Info

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Open cover of a compact metric space

Let $\mathcal{U}$ be an open cover of a compact metric space (X, d). Then there is a $\delta > 0$ called the **Lebesgue number for the covering** $\mathcal{U}$ such that every subset of X with diameter less than δ is contained in an element of the covering $\mathcal{U}$

$$A \subseteq X, \operatorname{diam}(A) < \delta \implies \exists U \in \mathcal{U} : A \subseteq U$$

- Let $\mathcal{U} = \{U_\alpha\}$ be an open cover of a compact metric space (X, d).
  - If $X \in \mathcal{U}$ the any positive $\delta > 0$ is a Lebesgue number for $\mathcal{U}$.
  - We assume $X \notin \mathcal{U}$ and choose a finite subcover $\{U_1, U_2, \ldots, U_n\}$ of $\mathcal{U}$.
  - We define the average

    $$f : X \to \mathbb{R}$$
    $$f(x) := \frac{1}{n}\sum_{i=1}^{n} d(x, X \setminus U_i)$$

    which means

    $$d(x, X \setminus U_i) \leq \max\{d(x, C_i) \mid 1 \leq i \leq n\}$$
    $$f(x) = \frac{1}{n}\sum_{i=1}^{n} d(x, X \setminus U_i) \leq \max\{d(x, C_i) \mid 1 \leq i \leq n\}$$

  - For each $x \in X$ we have $x \in U_i$ an ϵ-neighbourhood of x that lies in U_i. Then

    $$d(x, X \setminus U_i) \geq \epsilon$$

    which means

    $$f(x) = \frac{\epsilon}{n} + \underbrace{\ldots}_{\geq 0} > 0$$

  - Because X is compact and f is continuous

    $$f(X) \subseteq [\min f, \max f] \subset (0, \infty)$$

    and we define

    $$\delta := \min f > 0$$

  - Let $A \subseteq X$ has diameter less than δ and choose $x_0 \in A$. Then

    $$A \subseteq B_\delta(x_0)$$

  - Now

    $$\delta \leq f(x_0) \leq d(x_0, X \setminus U_m) := \max\{d(x_0, X \setminus U_i) \mid 1 \leq i \leq n\}$$

    This means

    $$\delta \leq d(x_0, X \setminus U_m)$$

    this means δ-neighborhood of x_0 is contained in the element U_m of the covering.

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - Open cover of a compact metric space

And it has 9 siblings.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - adherent, accumulation points, closure of a subset in a metric space
      - Balls in a metric space
      - Totally bounded subsets of a metric space
      - Compact subsets of a metric space
      - Covering of a subset of a metric space
      - Open cover of a compact metric space
      - Function between metric spaces
      - interior, exterior, boundary points in a metric space
      - open and closed sets in a metric space