Info

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Connected sets in $\mathbb{R}$

subset of $\mathbb{R}$ is connected iff it's an interval

- Let $I \subseteq \mathbb{R}$ be an interval or all of $\mathbb{R}$ and suppose it is not connected.
  - Then we have $I \subset U \cup V$ where U, V are non-empty open, $U \cap I, V \cap I \neq \emptyset$ and $U \cap V = \emptyset$.
  - Without loss of generality, choose $a \in U, b \in V$ with $a < b$ (otherwise change names of U, V).
  - Let

$$\begin{aligned} U_1 &:= U \cap I \cap [a, b] \\ V_1 &:= V \cap I \cap [a, b] \end{aligned}$$

    - We know U_1, V_1 are non-empty because they have a, b respectively. U_1, V_1 are disjoint because V, U are disjoint.
  - U_1 is bounded so $c := \sup U_1 < \infty$
  - ...
- If $S \subseteq \mathbb{R}$ is not an interval
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Current note has 0 direct children and 0 total descendants.

- stretchy
  - $(\mathbb{R}, +, \times, <, \sup)$
    - Connected sets in $\mathbb{R}$

And it has 3 siblings.

- stretchy
  - $(\mathbb{R}, +, \times, <, \sup)$
    - Connected sets in $\mathbb{R}$
    - Existence of $\mathbb{R}$
    - Metrics on $\mathbb{R}$