Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on June 20, 2023 5:46:43 PM,
was last modified on May 17, 2026 7:36:52 PM, and
was exported on September 7, 2026 3:31:13 PM.

Conservative 1-forms

Definition. Conservative 1-forms

A 1-form is **conservative** if for *any* picewise smooth, closed curve γ

$$\int_\gamma \omega = 0$$

It is such a form in the kernel of the integral map

$$\int : \Omega^\bullet(M) \to Z^1\mathrm{sing}^\bullet_\infty(M, \mathbb{R})$$

$$\int : H^1(\Omega^\bullet(M)) \cong H^1(\mathrm{sing}(M, \mathbb{R}))$$

integrals over non-closed curves

A 1-form ω is conservative $\iff$ for any two piecewise smooth curves(?segments) with same endpoints γ_1, γ_2

$$\int_{\gamma_1} \omega = \int_{\gamma_2} \omega$$

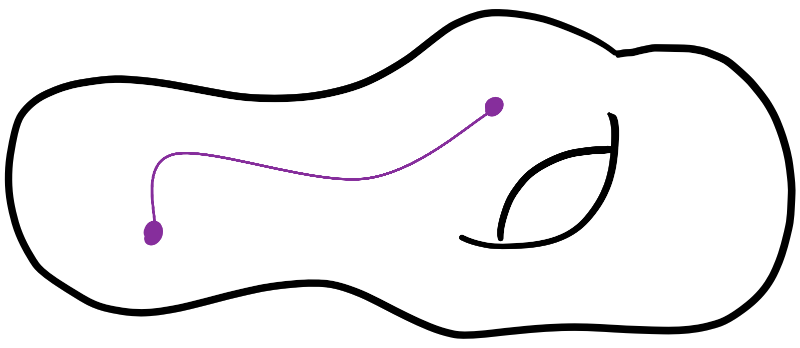
exactness

Definition. Potential function for a conservative 1-form on a connected smooth manifold

Assume M is a connected smooth manifold and a conservative 1-form ω. Choose a base-point $b_0 \in M$. Define a function $f_\omega : M \to \mathbb{R}$

$$f_\omega(q) := \int_{\gamma_{b_0 \to p}} \omega$$

where $\gamma_{b_0 \to p}$ is a piece-wise smooth curve joining b_0 and p.



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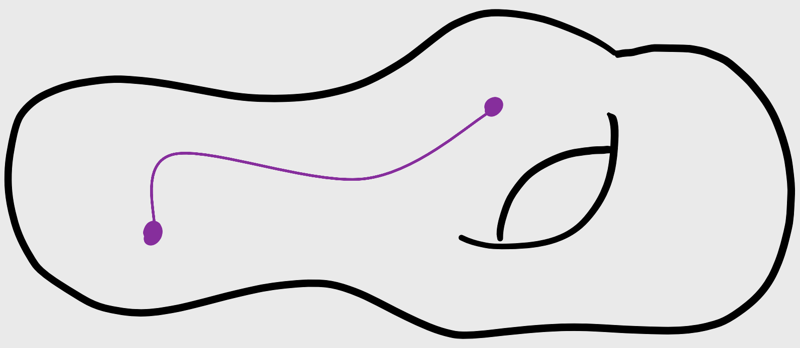

is a smooth function on M, such that

$$\mathrm{d}f_\omega = \omega \text{ on } M$$

When are closed differential 1-forms exact on a smooth manifold?

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Differential forms on smooth manifolds
          - Differential 1-forms on smooth manifolds
            - Conservative 1-forms

And it has 3 siblings.

- structures based on sets
  - manifolds
    - smooth $\mathbb{R}$-manifold
      - Tangents on a smooth manifold
        - Differential forms on smooth manifolds
          - Differential 1-forms on smooth manifolds
            - Closed differential 1-forms on smooth manifolds
            - When are closed differential 1-forms exact on a smooth manifold?
            - Conservative 1-forms