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Immersed submanifolds of a smooth manifold

Definition. Immersed submanifolds of smooth manifolds

Let M be a smooth manifold with or without boundary. An **immersed submanifold** of M is a subset $S \subseteq M$

- endowed with a topology $\mathfrak{T}_S$ (not necessarily the subspace topology)
- with respect to which it is a topological manifold without boundary
- and a smooth structure $\mathcal{A}_S$
- with respect to which the inclusion map

$$\iota : (S, \mathfrak{T}_S, \mathcal{A}_S) \hookrightarrow M$$

is a smooth immersion

Here, M is called the **ambient manifold** of S

- The **codimension** of S in M is $\dim M - \dim S$
- An **immersed hypersurface** is an embedded submanifold of codimension 1

(Images of injective immersions are immersed submanifolds) Let M be a smooth manifold with or without boundary, N is a smooth manifold and $F : N \rightarrow M$ is an **injective smooth immersion**. Then

$$\iota : (F(N), F_*\mathfrak{T}_N, F_*\mathcal{A}_N) \hookrightarrow M$$

is a **smooth immersed manifold** such that

$$F : N \rightarrow F(N)$$

is a diffeomorphism.

Definition. Immersed submanifolds of smooth manifolds with self-crossings are images $F(N)$ of smooth immersions

$$F : N \rightarrow M$$

which are non injective.

When is a smooth injective immersion an embedding

Let $F : N \rightarrow M$ is a smooth injective immersion.

$F(M)$ ↓	N is without boundary, connected, non-compact	N is with boundary, connected, non-compact
open subset	???	
compact, not a topological manifold	"figure 8 immersion"	
closed, non-compact subset	?	
not closed, not open	"dense torus curves"!	
is an embedded submanifold	$\implies F$ is an embedding?	

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