Info

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Types of isometries of $\mathbb{R}$-hyperbolic space

Definition. Types of isometries of $\mathbb{R}$-hyperbolic space

Let $\phi \in \mathrm{Isom}(\mathbb{R}\boldsymbol{H}^n)$. Then ϕ is said to be

- **elliptic** if ϕ fixes a point of $\mathbb{R}\boldsymbol{H}^n$
- **parabolic** if ϕ fixes no point of $\mathbb{R}\boldsymbol{H}^n$ and a unique point of $\partial\mathbb{R}\boldsymbol{H}^n$
- **hyperbolic** if ϕ fixes no point of $\mathbb{R}\boldsymbol{H}^n$ and two points of $\partial\mathbb{R}\boldsymbol{H}^n$

> **Proposition:**
>
> - A Mobius transformation of B^n is *elliptic* $\iff$ it is conjugate in $\mathrm{Moeb}(B^n)$ to an orthogonal transformation $O(n, \mathbb{R}) < \mathrm{Moeb}(B^n)$.
> - A Mobius transformation of $\mathbb{R}^n_+$ is *parabolic* $\iff$ it is conjugate in $\mathrm{Mob}(\mathbb{R}^n_+)$ to an an extension of an isometry of $\mathbb{R}^{n-1}$ $x \mapsto O(x) + b$ where $O \in O(n-1, \mathbb{R})$ and $b \in \mathbb{R}^{n-1}$.
> - A Mobius transformation of $\mathbb{R}^n_+$ is *hyperbolic* $\iff$ it is conjugate in $\mathrm{Mob}(\mathbb{R}^n_+)$ to an an extension of a linear map of $\mathbb{R}^{n-1}$ of the form $x \mapsto \lambda O(x)$ where $\lambda > 1$ and $O \in O(n-1, \mathbb{R})$.

[1]

Current note has 0 direct children and 0 total descendants.

- squishy
  - $\mathbb{R}\boldsymbol{H}^n$
    - Types of isometries of $\mathbb{R}$-hyperbolic space

And it has 5 siblings.

- squishy
  - $\mathbb{R}\boldsymbol{H}^n$
    - Hyperbolic ball model
    - Conic neighborhood
    - Hyperboloid model of $\mathbb{R}\boldsymbol{H}^n$
    - Hyperboloid model to ball model of $\mathbb{R}\boldsymbol{H}^n$
    - Types of isometries of $\mathbb{R}$-hyperbolic space

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1. John G. Ratcliffe - Foundations of Hyperbolic Manifolds (2006), p.151 ↩