Info

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$GL(n, \mathbb{C})$ is path-connected

> **Proposition:** The open set $GL(n, \mathbb{C}) \subset \mathbb{C}^{n^2}$ is path-connected.

Pick $A \in GL(n, \mathbb{C})$. Let

$$P(t) := \det(A + t(I - A))$$

- $P(0) = \det A \neq 0$ and $P(1) = \det I = 1 \neq 0$.
- As $P(t)$ is a polynomial the zero set is finite, so $\{P \neq 0\} \subset \mathbb{C}$ is path-connected. So we may pick a polygonal path

  $$\gamma : [0, 1] \to \{P \neq 0\} \subset \mathbb{C}$$

  such that

  $$\gamma(0) = 0 \in \{P \neq 0\}$$
  $$\gamma(1) = 1 \in \{P \neq 0\}$$

- Thus $P(\gamma(t)) \neq 0$ for all t by construction.
- Finally, we consider the path

  $$\Gamma(t) := A + \gamma(t)(I - A)$$

  where $\det \Gamma(t) = P(\gamma(t)) \neq 0$ so

  $$\Gamma : [0, 1] \to GL(n, \mathbb{C})$$

  such that

  $$\Gamma(0) = A, \Gamma(1) = I$$

- If $B_1, B_2 \in GL(n, \mathbb{C})$ we consider the path

  $$B_2 \Gamma(t)$$

  for Γ as above for $A := B_2^{-1} B_1$.

Current note has 0 direct children and 0 total descendants.

- squishy
  - GL
    - C n
      - $GL(n, \mathbb{C})$ is path-connected

And it has 2 siblings.

- squishy
  - GL
    - C n
      - $GL(n, \mathbb{C})$ is path-connected
      - surjectivity of exponential map onto $GL(n, \mathbb{C})$