Info

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Dual of Banach spaces

weak* topology

Definition. Weak* topology on dual $X^\star$ of a topological vector space

Let X be a topological k-vector space whose dual is $X^\star$. Then we have the evaluation maps

$$\begin{aligned} \mathrm{ev}_x : X^\star &\to k \\ \varphi &\mapsto \varphi(x) \end{aligned}$$

for each $x \in X$ which gives an injection

$$\begin{aligned} X &\hookrightarrow X^{\star *} \\ x &\mapsto \mathrm{ev}_x \end{aligned}$$

Assume these functionals separates points of $X^\star$. The X-topology[1] on $X^\star$ is called the **weak*-topology** on $X^\star$ under which topology

$$X^{\star\star} \cong X$$

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1. Let X be a k-vector space and $V \leq X^*$ be a vector space of separating linear functionals on X. Then the V-topology $\mathfrak{T}_V$ makes X into a locally convex space such that

   $$X^\star \cong V$$

   ↩

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- structures based on sets
    - Topological spaces
        - Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$
            - Banach spaces
                - Dual of Banach spaces

And it has 1 siblings.

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