

Info

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Sensitive dependence on initial conditions of dynamics of a map between metric space

Definition. Sensitive dependence on initial conditions of dynamics of a map between metric space

Let (X, d) be a [metric space](#) and $f : X \rightarrow X$ be a map. If there exists a $\epsilon > 0$ such that

$$\forall x \in X, \delta > 0 \left(\exists y \in X, n \in \mathbb{Z}_{>0} \left(\begin{array}{c} d(x, y) < \delta \\ d(f^n(x), f^n(y)) \geq \epsilon \end{array} \right) \right)$$

then dynamics of f is said to **exhibit ϵ -sensitive dependence on initial conditions**.

In other words, for every $\delta > 0$, $x \in X$ there exists $y \in X, n > 0$ such that x, y are δ -close but $f^n(x), f^n(y)$ are ϵ -far away, or even more compactly

$$\exists \epsilon \left(\forall x \in X, \delta > 0 \left(\exists y \in X, n \left(\begin{array}{c} d(x, y) < \delta \\ d(f^n(x), f^n(y)) \geq \epsilon \end{array} \right) \right) \right)$$

The negation is

$$\begin{aligned} \neg \epsilon > 0 \quad & \left(\forall x \in X, \delta > 0 \quad \left(\exists y \in X, n \quad \left(\begin{array}{c} d(x, y) < \delta \\ d(f^n(x), f^n(y)) \geq \epsilon \end{array} \right) \right) \right) \\ \forall \epsilon > 0 \quad & \left(\exists x \in X, \delta > 0 \quad \sim \left(\exists y \in X, n \quad \left(\begin{array}{c} d(x, y) < \delta \\ d(f^n(x), f^n(y)) \geq \epsilon \end{array} \right) \right) \right) \\ & \left(\exists x \in X, \delta > 0 \quad \left(\forall y \in X, n \quad \sim \left(\begin{array}{c} d(x, y) < \delta \\ d(f^n(x), f^n(y)) \geq \epsilon \end{array} \right) \right) \right) \\ & \left(\begin{array}{c} d(x, y) > \delta \\ d(f^n(x), f^n(y)) < \epsilon \end{array} \right) \right) \end{aligned}$$

Definition. ϵ -sensitive dependence and Liapunav stability

$f : X \rightarrow X$ exhibits ϵ-sensitive dependence at point $x \in X$ if there exists a number $\epsilon > 0$ such that $\forall \delta > 0, \exists (y_0 \in X, n \in \mathbb{Z}_{>0})$ where $d(x, y) < \delta, d(f^n x, f^n y) \geq \epsilon$	$f : X \rightarrow X$ is Liapunov stable at a point $x_0 \in X$ if for every $\epsilon > 0$ there exists a $\delta > 0$ so that $\forall n \geq 0, y_0 \in X, d(x, y) < \delta$ $\implies d(f^n x_0, f^n y_0) < \epsilon$
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can isometries exhibit a sensitive dependence on initial conditions?

A [metric space isometry](#) **cannot** exhibit a *sensitive dependence on initial conditions* >

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can contractions exhibit a sensitive dependence on initial conditions?

A [contraction map in a metric space](#) **cannot** exhibit a *sensitive dependence on initial conditions* >

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then dynamics of f is said to **exhibit ϵ -sensitive dependence on initial conditions**.

can continuous/discontinuous maps with fixed point exhibit a sensitive dependence on initial conditions?

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 - [Metric spaces](#)
 - [Metric spaces](#)
 - [Function between metric spaces](#)
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1. [sensitive_dep_initial_cond.pdf \(csulb.edu\)](#) ↩