Info

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Compact subsets of a metric space

A subset A of a metric space M is *sequentially compact*

Definition. Sequential compactness of a subset of a metric space

A subset A of a metric space M is **sequentially compact** if every sequence $(a_n) \subseteq A$ has a subsequence $(a_{n_k})_{k\in\mathbb{N}}$ that coverges to a limit in A.

$$\iff$$

it is *covering compact*

Definition. Covering compactness in a metric space

If every open covering of $A \subseteq M$ has a *finite subcovering* of A then A is **covering compact**.

Hence, we define a subset to be *compact* if it satisfies any if the above equivalent properties.

Definition. Compact subsets of a metric space

By their equivalence, a compact subset is defined by either of these definitions

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Definition. Covering compactness in a metric space

If every open covering of $A \subseteq M$ has a *finite subcovering* of A then A is **covering compact**.

- $K = \{x_1, \dots x_n\}$ be a finite set
  - Let $\{U_\alpha\}_{\alpha\in A}$ be any open covering of K
  - Then $\exists \alpha_i \in A$ such that

    $$x_i \in U_{\alpha_i}$$

    for all $1 \leq i \leq n$
  - Clearly $K \subseteq \bigcup_i U_{\alpha_i}$
  - Hence, $\{U_{\alpha_i}\}_{1\leq i\leq n}$ is a finite subcovering.

Every compact set of a metric space is closed and bounded.

Let (X, d) be a compact metric space and $f : X \to \mathbb{R}$ be a continuous function. Then f is bounded and f attains its maxima and minima.

Current note has 2 direct children and 2 total descendants.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - Compact subsets of a metric space
        - Covering compactness in a metric space
        - Sequential compactness of a subset of a metric space

And it has 9 siblings.

- structures based on sets
  - Metric spaces
    - Metric spaces
      - adherent, accumulation points, closure of a subset in a metric space
      - Balls in a metric space
      - Totally bounded subsets of a metric space
      - Compact subsets of a metric space
      - Covering of a subset of a metric space
      - Open cover of a compact metric space
      - Function between metric spaces
      - interior, exterior, boundary points in a metric space
      - open and closed sets in a metric space