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Fixed points of continuous endomorphisms of the n-disk

endomorphisms of the closed n-disk

(Brouwer's fixed point theorem) Let $\varphi : \overline{D^n} \to \overline{D^n}$ be a continuous map for $n \geq 2$, then it has a fixed point.

reducing to the smooth case

[1]

proof for the smooth case

Proof. The first part of the proof consists in reducing to the case where φ is a $\mathcal{C}^\infty$ map. We will not give the details here (see [45]). Next, starting from φ, which is assumed to be $\mathcal{C}^\infty$ and without fixed points, we construct a retraction

$$r : D^n \longrightarrow S^{n-1}$$

by sending $x \in D^n$ to the intersection point of the sphere S^{n-1} and the ray starting at $\varphi(x)$ and going through x. This way, if x is a point of the sphere, it stays in place. We thus have a $\mathcal{C}^\infty$ map r that restricts to the identity on the boundary. Sard's theorem asserts that this map has regular values. Let

$a \in S^{n-1}$ be one of them; then $r^{-1}(a)$ is a submanifold of dimension 1 of D^n, with boundary

$$\partial r^{-1}(a) = r^{-1}(a) \cap \partial D^n = \{a\}\,.$$

But a manifold of dimension 1 with boundary is diffeomorphic to a union of circles and closed intervals, so that its boundary consists of an even number of points. This gives a contradiction, and therefore the existence of a fixed point. □

combinatorial proofs

https://math.mit.edu/~fox/MAT307-lecture03.pdf

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And it has 1 siblings.

- stretchy
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1. Lafontaine, J.: Introduction aux Vari´et´es Diff´erentielles. Presses Universitaires de Grenoble, Grenoble (1996) ↩