Info

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Flow of a vector field in $\mathbb{R}^n$

Also the standard 1-order ODE in $\mathbb{R}^n$, the standard *ordinary differential equation*.

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title: geometric pov on differential equations: $1$-ODE in $\R^{n}$


| solving differential equations                     | analysis and
geometry of vector fields                                    |
| ----------------------------------------- | --------------------------
--------- |
| an equation $$\dot{x}=\mathbf{f}(x)$$                               |
a vector field $$\mathbf{f}:\mathcal{U} \subseteq \R^{n} \to \R^{n}$$
|
| solutions of the equation $$\begin{align} \dot{x}(t)&=\mathbf{f}(x(t))
\\ x(0)&=x_0 \end{align}$$                | integral curves of the
vector field $$t \mapsto \Phi^\mathbf{f}_{t}(x_{0})$$ |
| how solutions depend on initial conditions $$x_0 \mapsto x(t)$$ | flows
of the vector field    $$x_0 \mapsto \Phi^\mathbf{f}_{t}(x_0)$$
|
| conserved quantities                         | integrals of the vector
field      |
| (linearly) decoupling the differential equation $$\dot{z_{i}}= f_{i}
(z_i)$$ | (linear) coordinate transformation such that
$$\mathbf{f}=\sum_i f_i(z_{i}) \hat{z}_i$$ |
```

Definition. Flow of a vector field

Given a vector field $\mathbf{f} : \mathcal{P} \subseteq \mathbb{R}^n \to \mathbb{R}^n$ where $\mathcal{P}$ is called the *phase space*, a map $\Phi : \mathcal{D} \subseteq \mathbb{R} \times \mathcal{P} \to \mathbb{R}^n$

$$(t, \mathbf{x}) \mapsto \Phi_t(\mathbf{x})$$

is the **flow** of $\mathbf{f}$ if

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_t(x_0) = f(\Phi_t(x_0))$$

pov on differential equations: *convert n-degree differential equation in $\mathbb{R}$ to 1-ODE in $\mathbb{R}^n$*

From $f : \mathbb{R} \to \mathbb{R}$

$$\mathfrak{D}^n f(x) = \mathcal{F}(x, f(x), \mathfrak{D} f(x), \ldots, \mathfrak{D}^{n-1} f(x))$$

$$\downarrow$$

$q \in \mathbb{R}^{n+1}$ $q_i(x) := \mathfrak{D} f^n(x)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} q_0 \\ q_1 \\ \ldots \\ q_n \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ \ldots \\ \mathcal{F}(q_0, q_1, \ldots, q_{n-1}) \end{pmatrix}$$

Definition. Local and global solutions

Volume change by flows on $\mathbb{R}^n$

- ~~cat~~
- t - types/extension
  - types of $\mathcal{D}$
  - types of f
    - linear
    - polynomial
      - quadratic
    - rational
  - types of Φ_f
    - conservative/dissipative
- p - constructing more/attachments/constructed with
  - from the standard n-ODE in $\mathbb{R}$
  - vector fields
  - from derivatives of $\mathbb{R} \to G$
- m - structures/more structure
  - integrals Conserved quantities for Flow of a vector field in $\mathbb{R}^n$
  - k-integrable
  - symmetry

- v - properties space.R.n.f.field vector flows.v
    - local
        - existence, uniqueness
    - global
        - orbits
            - periodic
        - volumes/measure
        - Probability distribution of flow
    - parameters
        - bifurcations
- ~~s - subsets~~
- ~~f - functions/equations(polynomials, equations, differential equations)/dynamics~~
    - equivalent flows?
- b - function spaces, attachments, bundles and duals

- o - external effects
- e - equivalences
    - equivalences and automorphisms
    - identifications
- c - list/examples/classification upto isom

(Peano-Cauchy)

> Let f be continuous in
>
> > **Definition.** Flow of a vector field
> >
> > Given a vector field $\mathbf{f} : \mathcal{P} \subseteq \mathbb{R}^n \to \mathbb{R}^n$ where $\mathcal{P}$ is called the *phase space*, a map $\mathbf{\Phi} : \mathcal{D} \subseteq \mathbb{R} \times \mathcal{P} \to \mathbb{R}^n$
> >
> > $$(t, \mathbf{x}) \mapsto \mathbf{\Phi}_t(\mathbf{x})$$
> >
> > is the **flow** of **f** if
> >
> > $$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_t(x_0) = f(\Phi_t(x_0))$$
>
> then $\exists$ at least one local solution of the dynamical equation.

> Every solution $x(t)$ of
>
> > **Definition.** Flow of a vector field
> >
> > Given a vector field $\mathbf{f} : \mathcal{P} \subseteq \mathbb{R}^n \to \mathbb{R}^n$ where $\mathcal{P}$ is called the *phase space*, a map $\mathbf{\Phi} : \mathcal{D} \subseteq \mathbb{R} \times \mathcal{P} \to \mathbb{R}^n$
> >
> > $$(t, \mathbf{x}) \mapsto \mathbf{\Phi}_t(\mathbf{x})$$
> >
> > is the **flow** of **f** if
> >
> > $$\frac{\mathrm{d}}{\mathrm{d}t}\Phi_t(x_0) = f(\Phi_t(x_0))$$
>
> with $x(0) = x_0$ can be continued to a maximal level of existence $(t_1, t_2) \ni 0$.
> If t_1, t_2 are finite, then for any compact $\mathcal{K} \subseteq \mathcal{D}$, $\exists t \in (t_1, t_2)$ with $x(t) \notin \mathcal{K}$.

The last implication means that solutions will diverse or reach $\partial\mathcal{D}$.

> Children
>
> Dataview: No results to show for list query.

Current note has 0 direct children and 0 total descendants.

- stem
    - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
        - ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
            - Flow of a vector field in $\mathbb{R}^n$

And it has 10 siblings.

- stem
    - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_\Gamma\backslash^{\mathbb{R}^n}, _\Gamma\backslash^{\mathbb{C}^n}$
        - ODEs in $\mathbb{R}^n$ $\leftrightarrow$ Vector fields in $\mathbb{R}^n$
            - Constant of a flow in $\mathbb{R}^n$
            - Euler's iterative solution method for first order ODEs
            - Existence of integral curves of vector fields in $\mathbb{R}^n$
            - Fixed points
            - Flow of a vector field in $\mathbb{R}^n$
            - Graph $\to$ polynomial ODE
            - Gradient flows on $\mathbb{R}^n$
            - Hamiltonian vector fields in $\mathbb{R}^{2n}$ with standard symplectic form
            - Probability distribution of flow
            - Volume change by flows on $\mathbb{R}^n$