CW complexes

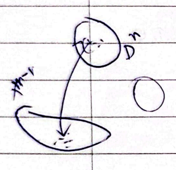
CW-complexes

* "π_2" will be affected by attaching D^3, but higher π_i's will not.

$\boxed{\text{CW-complexes (cell complexes)}}$

A CW-complex is a top space X, obtained by following procedure.

- we start by a discrete space X^0.
- If X^{n-1} has been constructed for $n \geq 1$, we take some cont. maps $\{f_\alpha : S^{n-1} \longrightarrow X^{n-1}\}$ and define X^n



$$X^n := X^{n-1} \coprod \left(\coprod_\alpha D^n \right) \Big/ \left\{ \begin{array}{l} \text{identifying } x \text{ and } f_\alpha(x) \\ \forall x \in S^{n-1}, \\ \text{and all } \alpha \end{array} \right\}.$$

* The space X^n is called the n-skeleton of X.

$$X := \bigcup_{n \in \mathbb{N}} X^n$$

CW complex!

(Top) A subset of X is defined to be open if its intersection with each X^n is open.

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* $X \xrightarrow{f \in e^1} Y$

~~~~ f induces homotopy group isom. $\not\Rightarrow$ X, & Y are homotopy equivalent.

A <u>relative CW complex</u> is a pair (X, A) where A is a subspace of X and X can be obtained by a similar procedure by taking $X_{-1} := A$.

A <u>CW pair</u> is a pair (X, A) where A is a subspace of X and they have compatible <u>CW-structures</u>.

(Thm) Given a connected CW-complex X, we have $\pi_1(X) = \pi_1(X^2)$.

2-skeleton.

proof X is obtained from X^2 by attaching higher dim cells.

// ? //

(i) ~~~~ Any loop in X and any homotopy of loops factors through X^n for some finite n. // [next time]

(ii) $\pi_1(X^n) = \pi_1(X^2)$

* Interior of D^n's are called <u>cells</u>.

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Types of topological spaces
      - CW complexes

And it has 9 siblings.

- structures based on sets
  - Topological spaces
    - Types of topological spaces
      - T_0 topological space
      - T_1 topological space
      - T_2 topological space or Hausdorff space
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- Compact Hausdorff spaces
- First countable topological space
- Second countable topological spaces
- Compact, metrizable, totally disconnected perfect topological spaces
- CW complexes
- Separable topological spaces