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Open subsets of $\mathbb{R}^n$ with $\mathcal{C}^2$ boundary given by a function

Let $\Omega \subseteq \mathbb{R}^n$ be an open subset such that there exists $\rho \in \mathcal{C}^2(\overline{\Omega})$ such that

$$\begin{aligned}\Omega &= \{\rho < 0\} \\ \mathrm{d}\rho &\neq 0 \text{ on } \partial\Omega\end{aligned}$$

For $x \in \partial\Omega$, we define

$$T_x\partial\Omega := \ker \mathrm{d}_x\rho$$

Proposition??: Ω is convex $\iff (T_x\partial\Omega + x) \cap \Omega = \emptyset \iff$

$$\forall \xi \in T_x\partial\Omega, \sum_{j,k=1}^n \frac{\partial^2 \rho}{\partial x_j \partial x_k} \Big|_x \xi_j \xi_k \geq 0$$

[1]

Current note has 0 direct children and 0 total descendants.

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And it has 39 siblings.

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1. [Herbert Alexander, John Wermer - Several Complex Variables and Banach Algebras \(1998, Springer\).](#), p.205 ↩