> [!info] Info
>
> This note, that was found here, as a part of a collection
> is written (completely with human hands) by Rupadarshi Ray,
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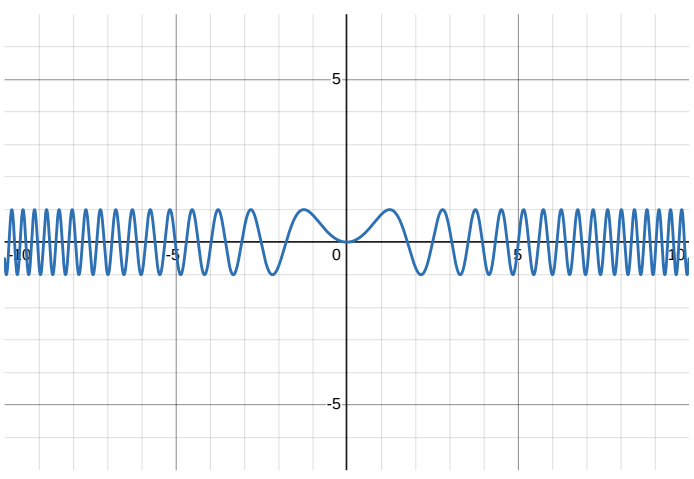
$$\sin x^2$$

#function/real/1

- $$\begin{aligned}\mathbb{R} &\to [-1,1]\\ x &\mapsto \sin x^2\end{aligned}$$

  - It is smooth (analytic even), bounded by 1.



- **Proposition:** The smooth function $\sin x^2$ is **not** uniformly continuous on $\mathbb{R}$, that is, for $\epsilon = 1$

  $$\forall \delta > 0 \exists x, y \in \mathbb{R} : |x-y| < \delta \text{ and } \left|\sin x^2 - \sin y^2\right| \geq 1$$

  - Choose

    $$\begin{aligned}x_n &:= \sqrt{n\pi}\\ y_n &:= \sqrt{n\pi + \frac{\pi}{2}}\end{aligned}$$

    so that

    $$\begin{aligned}\sin x_n^2 &= 0\\ \sin y_n^2 &= 1\end{aligned}$$

  - Then

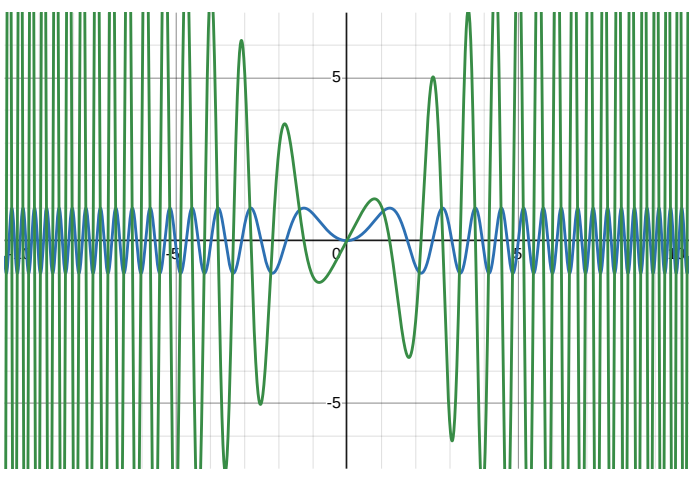
    $$|x_n - y_n| < \frac{1}{\sqrt{n}}$$

  - For any $\delta > 0$ choose $n > \delta^{-2}$, then we have

    $$\begin{aligned}|x_n - y_n| &< \delta\\ \text{but } \left|\sin x_n^2 - \sin y_n^2\right| &\geq 1\end{aligned}$$

derivative

- $$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x^2) = 2x\cos(x^2)$$



- - $$\frac{\mathrm{d}}{\mathrm{d}x}\sin(x^2) = 2x\cos(x^2)$$

  is unbounded on $\mathbb{R}$ as for all $x_n^2 = 2n\pi$ as

  $$2x_n \underbrace{\cos(x_n)}_{1} = 2x_n$$

  - So $\sin(x^2)$ is a bounded function on $\mathbb{R}$ with unbounded derivative

anti-derivative

Fresnel integral - Wikipedia

Current note has 0 direct children and 0 total descendants.

- squishy
  - pow
    - $\sin x^2$

And it has 28 siblings.

- squishy
  - pow
    - Angle function

- Bessel functions
- $$\cos \sqrt{z}$$
- The complex logarithm
- $\sum_{n \geq 0} z^{n!}$
- $z^3 - 3z$
- $$\frac{r_0}{1 + \epsilon \cos b\theta}$$
- $\cos\left(\pi \frac{p}{q}\right)$
- $\frac{\cos x^k}{x}$
- Elliptic integrals and Jacobi elliptic functions
- The complex exponential $\exp(z)$
- $\exp\left(\frac{z+1}{z-1}\right)$
- e^{-x^2}
- [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.]]$
- $\int \frac{1}{x^2+bx+c}$
- $\frac{\exp(z)}{1-z}$
- $\sin x$
- $\sin(x \sin(x))$
- $\frac{\sin x}{x}$
- stock.sin x by xk
- $\sin x^2$
- $\sqrt{z}$
- $\sqrt{z^2 - 1}$
- Theta functions
- Weierstrass elliptic function $\wp$
- Weierstrass' primary factors
- $x^2 \sin \frac{1}{x}$
- $x + x^2 \sin \frac{1}{x}$