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Absolutely continuous functions on $[a, b] \leftrightarrow \int_{[a, -]} (L^1[a, b])$

Definition. Absolutely continuous function on $[a, b]$

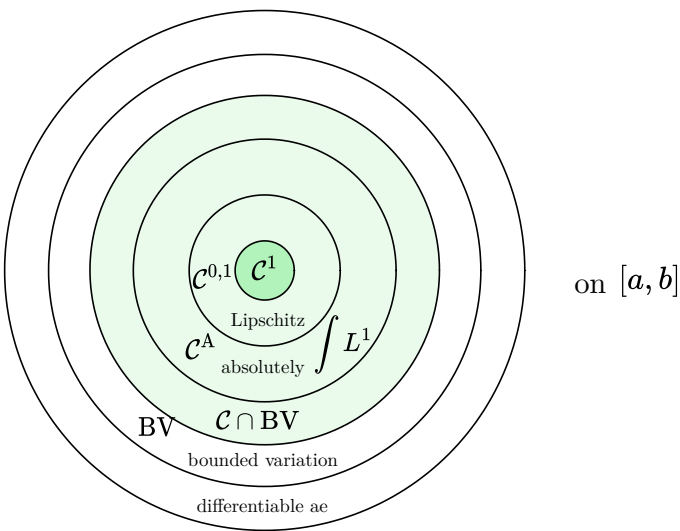
A function F defined on $[a, b]$ is **absolutely continuous** if for any $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ so that for every collection $(a_k, b_k) \subseteq [a, b]$ of disjoint intervals $1 \leq k \leq N$ we have

$$\sum_{k=1}^n (b_k - a_k) < \delta(\epsilon) \implies \sum_{k=1}^n |F(b_k) - F(a_k)| < \epsilon$$

The space of all absolutely continuous functions on $[a, b]$

$$\mathcal{C}^A[a, b]$$

may equipped with the supremum $\|\cdot\|_\infty$ or variation V norms.



$$\int_{[a, -]} : L^1[a, b] \rightarrow \mathcal{C}^A[a, b]$$

- Let $f \in L^1[a, b]$ and consider

$$F(x) := \int_{[a, x]} f$$

- Then

$$\begin{aligned} F(x) - F(y) &= \int_{[a, x]} f - \int_{[a, y]} f \\ &= \int_{[x, y]} f \\ \implies |F(x) - F(y)| &\leq \int_{[x, y]} |f| \end{aligned}$$

- By

(Absolute continuity of f dm) Let $f \in L^1(\mathbb{R}^d)$. Then for all $\epsilon > 0$ there exists $\delta_\epsilon > 0$ such that for every measurable set E such that $m(E) < \delta_\epsilon$

$$\int_E |f| < \epsilon$$

we have $\forall \epsilon > 0 \exists \delta(\epsilon) > 0$ such that for every

$$I = \bigsqcup_k (a_k, b_k)$$

such that

$$m(I) = \sum_k (b_k - a_k) < \delta(\epsilon)$$

the integral

$$\int_I |f| < \epsilon$$

- Here,

$$\begin{aligned} |F(b_k) - F(a_k)| &\leq \int_{[a_k, b_k]} |f| \\ \implies \sum_k |F(b_k) - F(a_k)| &\leq \sum_k \int_{[a_k, b_k]} |f| \\ &= \int_I |f| \\ &< \epsilon \end{aligned}$$

$$\mathfrak{D} : \mathcal{C}^A[a, b] \rightarrow L^1[a, b]$$

(Fundamental theorem of analysis, in the almost everywhere sense) A function

$$F : [a, b] \rightarrow \mathbb{R}$$

is *absolutely continuous* on $[a, b] \implies F'$ exists m -almost everywhere and is L^1 . In that case,

$$F(x) = F(a) + \int_{[a, x]} F' \quad \text{on } [a, b]$$

Moreover $F' = 0$ m -almost everywhere $\implies F$ is constant m -almost everywhere.
Conversely, for every $f \in L^1[a, b]$ there exists a function F which is differentiable m -almost everywhere and

$$F' = f \text{ ae}$$

and in fact we may take F to be the absolutely continuous function $x \mapsto c_0 + \int_{[a, x]} f$ for any $c_0 \in \mathbb{C}$.

Current note has 0 direct children and 0 total descendants.

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And it has 10 siblings.

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