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Cocompact Fuchsian groups

Definition. Signature of a cocompact Fuchsian group

Let Γ be a cocompact Fuchsian group with signature $(g; m_*)$. Then

$$\mu(D(\Gamma)) = 2\pi \left(2g - 2 + \sum_* \left(1 - \frac{1}{m_*} \right) \right)$$

Let the signature of Γ be $(g; m_1, \dots, m_r)$ Consider the convex polygon $D(\Gamma)$ which has finitely many sides.

- We introduce new vertices on sides of D which

$$S := \text{vertex}(D(\Gamma)) \cup \{z \in \text{mid}(\text{sides}(D(\Gamma))) \mid |\Gamma_z| = 2\}$$

- For all $z \in D(\Gamma, z_0)$ we have

$$|\Gamma \{z\} \cap \partial D_\Gamma(z_0)| < \infty$$

and

$$\sum_{z \in \Gamma \{z\} \cap \partial D_\Gamma(z_0)} \angle_z D_\Gamma(z_0) = \frac{2\pi}{|\Gamma_z|}$$

- In $D(\Gamma)$ there are r Γ -orbits of S -vertices with stabilizers of size $m_1, \dots, m_r$. The sum of angles of those

$$\frac{2\pi}{m_1} + \dots + \frac{2\pi}{m_r}$$

- There are s Γ -orbits of S -vertices whose stabilizers are trivial. The sum of angles of those is

$$2\pi s$$

- By

Let $P \subset H^2_\mathbb{U}$ be a convex polygon with n sides and internal angles $\theta_1, \dots, \theta_n$. Then

$$\mu(P) = (n - 2)\pi - \sum_{j=1}^n \theta_j$$

the area

$$\mu(D(\Gamma)) = (n - 2)\pi - 2\pi s - 2\pi \left(\sum_i \frac{1}{m_i} \right)$$

- The quotient $\Gamma \backslash H^2$ is of genus g , and has a CW-complex structure with $r + s$ vertices and $n/2$ edges

$$\begin{aligned} 2 - 2g &= \chi \left(\Gamma \backslash H^2 \right) = r + s - \frac{n}{2} + 1 \\ \implies n &= 4g - 2 + 2(r + s) \end{aligned}$$

- This means the area is

$$\begin{aligned} \mu(D(\Gamma)) &= (4g - 4)\pi + 2\pi(r + s) - 2\pi s - 2\pi \sum_i \frac{1}{m_i} \\ &= 2\pi(2g - 2) + 2\pi \sum_i \left(1 - \frac{1}{m_i} \right) \end{aligned}$$

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And it has 17 siblings.

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