Info

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Cocompact Fuchsian groups

Definition. Signature of a cocompact Fuchsian group

Let Γ be a cocompact Fuchsian group with signature $(g; m_*)$. Then

$$\mu(D(\Gamma)) = 2\pi\left(2g - 2 + \sum_* \left(1 - \frac{1}{m_*}\right)\right)$$

Let the signature of Γ be $(g; m_1, \ldots, m_r)$ Consider the convex polygon $D(\Gamma)$ which has finitely many sides.

- We introduce new vertices on sides of D which

$$S := \operatorname{vertex}(D(\Gamma)) \cup \{z \in \operatorname{mid}(\operatorname{sides}(D(\Gamma))) | |\Gamma_z| = 2\}$$

- For all $z \in D(\Gamma, z_0)$ we have

$$|\Gamma\{z\} \cap \partial D_\Gamma(z_0)| < \infty$$

and

$$\sum_{z \in \Gamma\{z\} \cap \partial D_\Gamma(z_0)} \angle_z D_\Gamma(z_0) = \frac{2\pi}{|\Gamma_z|}$$

- In $D(\Gamma)$ there are r Γ-orbits of S-vertices with stabilizers of size $m_1, \ldots, m_r$. The sum of angles of those

$$\frac{2\pi}{m_1} + \cdots + \frac{2\pi}{m_r}$$

- There are s Γ-orbits of S-vertices whose stabilizers are trivial. The sum of angles of those is

$$2\pi s$$

- By

Let $P \subset H^2_{\mathbb{U}}$ be a convex polygon with n sides and internal angles $\theta_1, \ldots, \theta_n$. Then

$$\mu(P) = (n-2)\pi - \sum_{j=1}^{n} \theta_j$$

the area

$$\mu(D(\Gamma)) = (n-2)\pi - 2\pi s - 2\pi\left(\sum_i \frac{1}{m_i}\right)$$

- The quotient $\Gamma \backslash H^2$ is of genus g, and has a CW-complex structure with $r+s$ vertices and $n/2$ edges

$$2 - 2g = \chi\left(\Gamma \backslash H^2\right) = r + s - \frac{n}{2} + 1$$
$$\implies n = 4g - 2 + 2(r+s)$$

- This means the area is

$$\begin{aligned}\mu(D(\Gamma)) &= (4g-4)\pi + 2\pi(r+s) - 2\pi s - 2\pi\sum_i \frac{1}{m_i} \\ &= 2\pi(2g-2) + 2\pi\sum_i\left(1 - \frac{1}{m_i}\right)\end{aligned}$$

- Decomposition of hyperbolic surfaces
- Infinitely generated Fuchsian groups
- Limit set of Fuchsian groups
- Marked hyperbolic surfaces
- subst.R PSL 2 discrete subg.over Q-bar
- Fuchsian Schottky groups
- Spectrum of hyperbolic surfaces
- Torsion-free Fuchsian groups
- Cocompact torsion-free Fuchsian groups
- Fuchsian triangle groups