

Info

This note, that [was found here](#), as a part of [a collection](#) is written (completely with human hands) by [Rupadarshi Ray](#), was created on December 26, 2024 7:29:48 PM, was last modified on September 7, 2026 1:11:00 PM, and was exported on September 7, 2026 3:09:54 PM.

Fixed points of functions

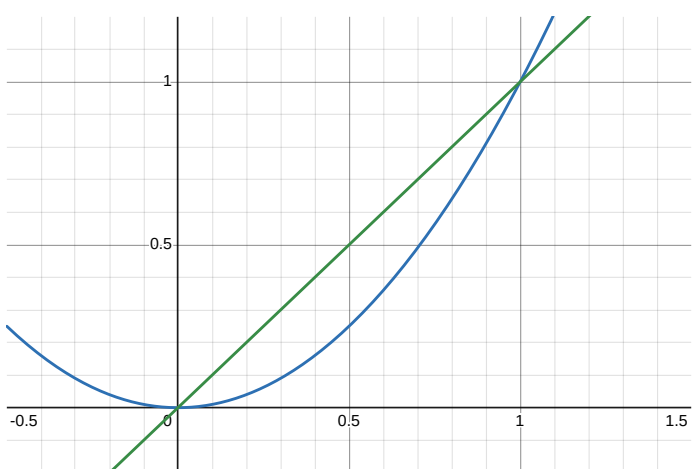
$U \subseteq \mathbb{R} \rightarrow U$

fixed points of continuous functions

$[a, b] \rightarrow [a, b]$

Example

x^2

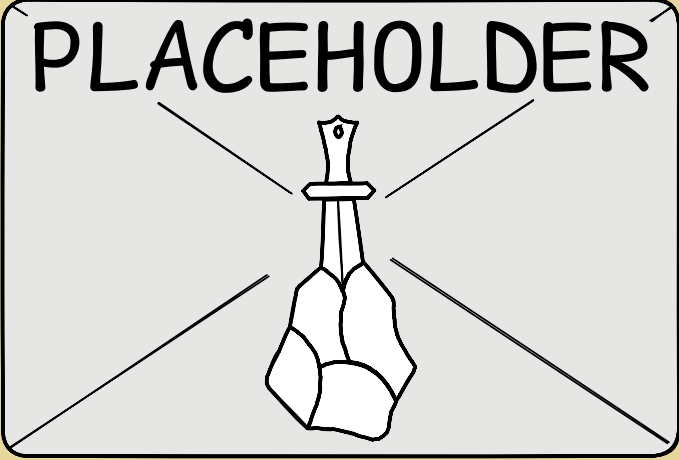


Let

$f : [a, b] \rightarrow [a, b]$

be a continuous function. Then there is a point $x \in [a, b]$

PLACEHOLDER



such that

$f(x) = x$

that is, a **fixed point**.

- A continuous function

$$f : [a, b] \rightarrow [a, b]$$

means

$$f(a), f(b) \in [a, b]$$

Then consider the continuous function

$$\begin{aligned} [a, b] &\rightarrow \mathbb{R} \\ x &\mapsto f(x) - x \end{aligned}$$

- This function maps

$$\begin{aligned} a &\mapsto f(a) - a \geq 0 \\ b &\mapsto f(b) - b \leq 0 \end{aligned}$$

- If both values are not equal, by

(Intermediate value theorem for continuous $[a, b] \rightarrow \mathbb{R}$)

Let $f : [a, b] \rightarrow \mathbb{R}$ be continuous and suppose there are two points $\alpha < \beta$ in $[a, b]$ such that $f(\alpha) \neq f(\beta)$. Then $f(x)$ takes every value between $f(\alpha)$ and $f(\beta)$ for $x \in (\alpha, \beta)$. That is

$$f(\alpha, \beta) \supseteq (f(\alpha), f(\beta))$$

we have a point $c \in [a, b]$ where

$$f(c) - c = 0$$

- If both values are equal, then both are zero, so the function $f(x) = x$ on $[a, b]$.

Current note has 0 direct children and 0 total descendants.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [R1](#)
 - [Fixed points of functions \$U \subseteq \mathbb{R} \rightarrow U\$](#)

And it has 5 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
 - [R1](#)
 - [Types of discontinuities of functions on \$\mathbb{R}\$](#)
 - [Extremum of functions \$\mathbb{R} \rightarrow \mathbb{R}\$](#)
 - [Fixed points of functions \$U \subseteq \mathbb{R} \rightarrow U\$](#)

- $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ of functions $f, g : U \subseteq \mathbb{R} \rightarrow \mathbb{R}$
- Total variation and sequence of continuous functions