Info

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Double derivative/Laplace operator on the circle

We identify

$$\begin{aligned}\mathbb{R} &\to S^1\\ x &\mapsto e^{2\pi i n x}\end{aligned}$$

so

$$L^p[0,1] = L^p(S^1)$$

and

$$\{f \in \mathcal{C}^k[0,1] \mid f(0) = f(1)\} = \mathcal{C}^k(S^1)$$

Definition. Space of continuously double differentiable functions on S^1 with boundary value 0

$$\begin{aligned}\mathfrak{D}^2 : \mathcal{C}^2(S^1) &\to \mathcal{C}^0(S^1)\\ f &\mapsto f''\end{aligned}$$

> **Proposition:** The set of functions
>
> $$\{e^{2\pi i n x} \mid n \in \mathbb{Z}\}$$
>
> is a set of orthogonal basis of $L^2[0,1]$.

- In this basis, the derivative operator is

$$\mathfrak{D} = \begin{bmatrix} \ddots & & & & & & \\ & 0 & & & & & \\ & & 2\pi i & & & & \\ & & & 6\pi i & & & \\ & & & & \ddots & & \\ & & & & & 2\pi i n & \\ & & & & & & \ddots \end{bmatrix}$$

- In this basis, the double derivative operator is

$$\mathfrak{D}^2 = \begin{bmatrix} \ddots & & & & & & \\ & 0 & & & & & \\ & & -(2\pi)^2 & & & & \\ & & & -(6\pi)^2 & & & \\ & & & & \ddots & & \\ & & & & & -(2\pi n)^2 & \\ & & & & & & \ddots \end{bmatrix}$$

exponentiating the double derivative

From

> - In this basis, the double derivative operator is
>
> $$\mathfrak{D}^2 = \begin{bmatrix} \ddots & & & & & & \\ & 0 & & & & & \\ & & -(2\pi)^2 & & & & \\ & & & -(6\pi)^2 & & & \\ & & & & \ddots & & \\ & & & & & -(2\pi n)^2 & \\ & & & & & & \ddots \end{bmatrix}$$

we have

$$\exp(t\mathfrak{D}^2) = \begin{bmatrix} \ddots & & & & & & \\ & 1 & & & & & \\ & & e^{-(2\pi)^2 t} & & & & \\ & & & e^{-(6\pi)^2 t} & & & \\ & & & & \ddots & & \\ & & & & & e^{-(2\pi n)^2 t} & \\ & & & & & & \ddots \end{bmatrix}$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $_{\Gamma}\backslash^{\mathbb{R}^n}, _{\Gamma}\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Double derivative/Laplace operator on the circle

And it has 10 siblings.

- stem

- subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash\mathbb{R}^n, \Gamma\backslash\mathbb{C}^n$
  - Differentiable functions
    - Functions $(a, b) \to \mathbb{R}$ differentiable at a point
    - Comparing a function and it derivative
    - Absolutely continuous functions on $[a, b]$ $\leftrightarrow$ $\int_{[a,-]}(L^1[a, b])$
    - Distributional derivatives
    - Double derivative/Laplace operator on the circle
    - Fractional derivative
    - Infinite limit of derivatives
    - Space of continuous and continuously differentiable functions on $\mathbb{R}$
    - Derivative of maps $\mathbb{R}^n \to \mathbb{R}^m$
    - Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$