Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on September 6, 2026 9:06:49 PM,
was last modified on September 6, 2026 9:49:51 PM, and
was exported on September 7, 2026 3:24:54 PM.

Image of balls under linear map between topological vector spaces

Let $T : X \to Y$ be a bounded linear map between normed linear spaces X, Y. Then for any $x \in X, r > 0$ we have

$$\sup_{z \in B(x,r)} \|T(z)\| \geq \|T\| r$$

- $$\begin{aligned} \max\{\|T(x-\xi)\|, \|T(x+\xi)\|\} &\geq \frac{\|T(x-\xi)\| + \|T(x-\xi)\|}{2} \\ &\geq \frac{1}{2}\|T(x-\xi) + T(x-\xi)\| \\ &= \frac{1}{2}\|2T(\xi)\| \\ &= \|T(\xi)\| \end{aligned}$$

  - Now

$$\sup_{\xi \in B(0,r)} \|T(x+\xi)\| \geq \sup_{\xi \in B(0,r)} T(\xi) = \|T\| r$$

https://www.desmos.com/calculator/96zlnlwesx

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Topological spaces
    - Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$
      - map
        - Image of balls under linear map between topological vector spaces

And it has 3 siblings.

- structures based on sets
  - Topological spaces
    - Topological vector spaces over $\mathbb{R}$ or $\mathbb{C}$
      - map
        - Image of balls under linear map between topological vector spaces
        - Limit of a sequence of linear maps
        - Uniform boundedness of a famly of linear maps