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Thurston geometries

 Let $G \curvearrowright X$ be a **transitive** and smooth Lie group action. Then X admits a G -invariant Riemannian metric $\iff$ for some (hence all) $x \in X$ the image

$$G_x \rightarrow GL(T_x X)$$

has compact closure $\iff$ the image is conjugate to a subgroup of $O(T_x X)$.

 Definition. Thurston geometries

A **Thurston geometry** is a smooth *transitive, faithful* Lie group action $G \curvearrowright X$ on a *simply connected* smooth manifold X such that

- point stabilizers $\text{stab}_G(x) < G$ is *compact* for one, hence all, $x \in X$
- the $G \curvearrowright X$ action is *maximal* among smooth transitive Lie group actions on X with compact point stabilizers
- there exists *at least one* compact $G \curvearrowright X$ -manifold

So, we have

$$G \curvearrowright G/K \cong_{\text{Man}} X$$

where $K := \text{stab}(x)$ is compact.

And as X is simply connected, the $G \curvearrowright X$ -manifolds are quotients of X by discrete subgroups of G ?

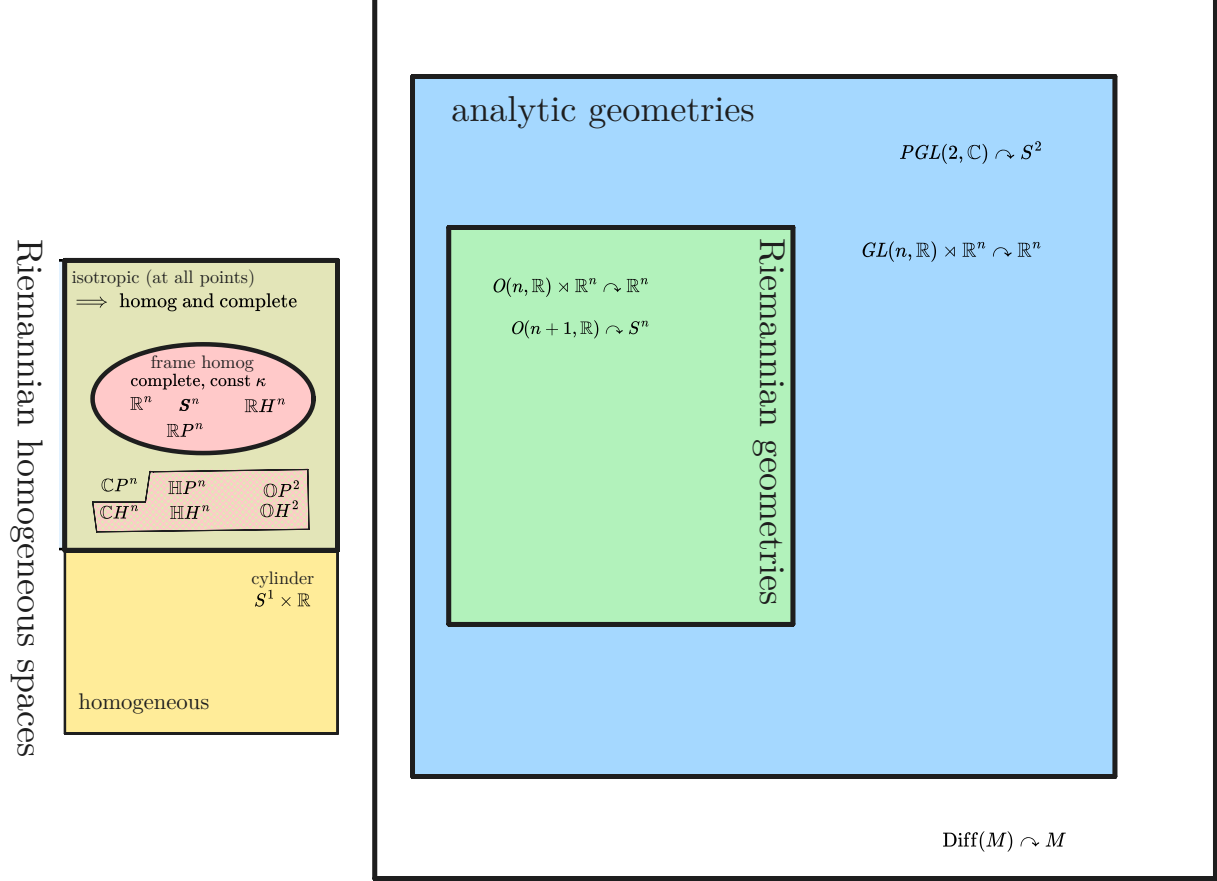
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Proposition: Thurston geometries are necessarily of the form

$$(\text{Isom}(X, g), X)$$

for some simply connected X and in particular *analytic*.

We require *Riemannian geometry* to be made of the maximal group of diffeomorphisms possible, however *analytic geometries* do not/cannot have such constraints.



[1]

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1. https://web.archive.org/web/20230104202309/https://www.cmi.ac.in/~vijayr/nachiketa_thurston.pdf

