Info

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Polynomial maps $\mathbb{C}^n \to \mathbb{C}^n$ of degree at most d

Let

$$\mathbb{C}[\mathbb{C}^n]_{\leq d}$$

be the space of d variable polynomials over $\mathbb{C}$ of degree at most 3.

The space of polynomial maps $\mathbb{C}^n \to \mathbb{C}^n$ is of degree at most d is

$$\mathrm{EndPoly}(\mathbb{C}^n)_{\leq d} = \mathbb{C}[\mathbb{C}^n]^n_{\leq d}$$

The Jacobian determinant gives a polynomial map

$$\mathrm{EndPoly}(\mathbb{C}^n)_{\leq d} \xrightarrow{\det \mathfrak{D}} \mathbb{C}[X_1, \ldots, X_n]_{\leq D}$$

The subset of polynomial automorphisms

$$\mathrm{AutPoly}(\mathbb{C}^n) \subseteq (\det \mathfrak{D})^{-1}(\mathbb{C} \setminus \{0\})$$

subset of automorphisms of Jacobian determinant 1

$$\mathrm{AutPoly}(\mathbb{C}^n)^1_{\leq d} \subseteq \mathbb{C}[\mathbb{C}^n]^n_{\leq d}$$

is Zariski closed.

[1]

subset of endomorphisms of Jacobian determinant 1

Let $Y \subseteq (\det \mathfrak{D})^{-1}(1)$ be an irreducible component. Then either $Y \subseteq \mathrm{AutPoly}(\mathbb{C}^n)$ or a generic element of Y is not injective.

Question

Is $(\det \mathfrak{D})^{-1}(1)$ irreducible?

Current note has 0 direct children and 0 total descendants.

- squishy
  - Polynomials
    - Polynomial maps $\mathbb{C}^n \to \mathbb{C}^n$ of degree at most d

And it has 17 siblings.

- squishy
  - Polynomials
    - $X^n + Y^n = Z^n$
    - $8X^3 + 4X^2 - 4X - 1$
    - $x^2 + y^2 - 3$
    - Folium of Descartes

      $$X^3 + Y^3 - 3aXY$$

    - $X^3 + Y^3 - a$
    - $3X^3 + 10X^2 + 3 - Y^2$
    - $3X^8 + 10X^4 + 3 - Y^2$
    - $XYZ - 1$
    - Pythagorean triples
    - Bolza curve
    - Irreducible polynomials in $\mathbb{C}[x, y]$
    - Polynomial maps $\mathbb{C}^n \to \mathbb{C}^n$ of degree at most d
    - Chebyshev polynomials of the first kind
    - Logistic map
    - Dynamics of the map $z^2 + c$
    - $\begin{pmatrix} x \\ y \\ y \end{pmatrix} \mapsto \begin{pmatrix} (1+xy)^3 z + y^2(1+xy)(4+3xy) \\ y + 3x(1+xy)^2 z + 3xy^2(4+3xy) \\ 2x - 3x^2 y - x^3 z \end{pmatrix}$
    - $(w^2 - 1)^2$

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1. https://arxiv.org/pdf/2607.20597 ↩