Info

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Pre-compact subsets of $(\mathrm{EndTop}(X, Y), \mathfrak{T}_{\mathrm{cpt}})$

Let X be a topological space, (Y, d) be a metric space, and $\mathcal{F} \subseteq \mathrm{EndTop}(X, Y)$ be a family of continuous maps

$$\mathcal{F} \ni f : X \to Y$$

Definition. Equicontinuous families in $\mathrm{EndTop}(X, Y)$

Let X be a topological space, (Y, d) be a metric space, and $\mathcal{F} \subseteq \mathrm{EndTop}(X, Y)$. Then

- $\mathcal{F}$ is **equicontinuous at** $x_0 \in X$ if $\forall \epsilon > 0$ there exists an open nb $x_0 \in U \subseteq X$ such that

$$\begin{aligned} &\forall x \in U, \forall f \in \mathcal{F} \;\; d(f(x), f(x_0)) < \epsilon \\ \iff &\forall z \in \mathcal{F}(U), d(z, f(x_0)) < \epsilon \end{aligned}$$

- $\mathcal{F}$ is **equicontinuous** if it is equicontinuous at all points $x \in X$.

equicontinuous and pointwise precompact families are pre-compact

(weak Arzela-Ascoli theorem onto metric spaces) Let X be a topological space, (Y, d) be a metric space, and

$$\mathcal{F} \subseteq (\mathrm{EndTop}(X, Y), \mathfrak{T}_{\mathrm{cpt}})$$

If $\mathcal{F}$ is equicontinuous and pointwise precompact then $\mathcal{F}$ is **pre-compact**.
If X is locally compact and Hausdorff, then converse also holds.

equicontinuous and pointwise precompact are sequentially pre-compact on σ-compact and locally compact spaces

(sequential Arzela-Ascoli theorem on compact spaces onto metric spaces) Let X be *compact* and (Y, d) be a metric space. Then for a equicontinuous and pointwise bounded

$$\mathcal{F} \subseteq (\mathrm{EndTop}(X, Y), \mathfrak{T}_{\mathrm{cpt}})$$

any sequence in $\mathcal{F}$ always has a subsequence in $\mathcal{F}$ that converges uniformly on X to a continuous map $X \to Y$.

By

(weak Arzela-Ascoli theorem onto metric spaces) Let X be a topological space, (Y, d) be a metric space, and

$$\mathcal{F} \subseteq (\mathrm{EndTop}(X, Y), \mathfrak{T}_{\mathrm{cpt}})$$

If $\mathcal{F}$ is equicontinuous and pointwise precompact then $\mathcal{F}$ is **pre-compact**.
If X is locally compact and Hausdorff, then converse also holds.

and

> **Proposition:** If X is *compact* and (Y, d) is any metric space, then the compact-open topology on Y^X and the topology of uniform convergence coincides
>
> $$\mathfrak{T}_{\mathrm{cpt}} = \mathfrak{T}_{\mathrm{uniform}}$$

(sequential Arzela-Ascoli theorem on σ-compact and locally compact spaces onto metric spaces) Let X be a σ-compact and locally compact topological space and (Y, d) be a metric space. Then a equicontinuous and pointwise bounded subset

$$\mathcal{F} \subseteq (\mathrm{EndTop}(X, Y), \mathfrak{T}_{\mathrm{cpt}})$$

is sequentially pre-compact (any sequence in $\mathcal{F}$ always has a subsequence in $\mathcal{F}$ that converges uniformly on compact subsets of X continuous map $X \to Y$).

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And it has 1 siblings.