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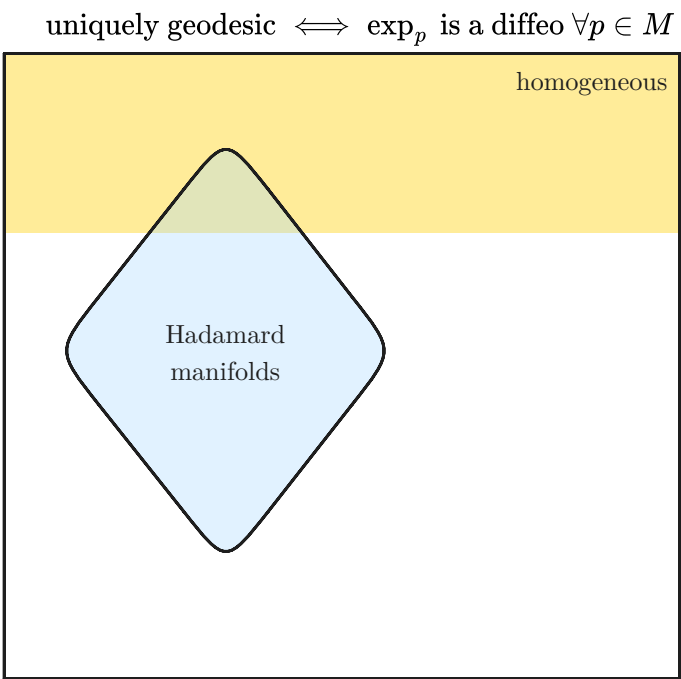
Uniquely geodesic Riemannian homogeneous manifolds

Proposition: Let (M, g) be a complete Riemannian manifold. Then the following are equivalent

- 1. every geodesic in (M, g) is (globally) **length-minimizing**
- 2. every unit speed geodesic in (M, g) is a minimal geodesic
- 3. every pair of points in (M, g) is joined by a **unique** minimal geodesic segment
- 4. the exponential

$$\exp_p : T_p M \rightarrow M$$

is a diffeomorphism for all $p \in M$



Simply connected, homogeneous *Cartan-Hadamard manifolds* are examples of such Riemannian manifolds. ^[1]

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1. [riemannian geometry - A Converse to Cartan–Hadamard theorem? - MathOverflow](#) $\leftrightarrow$