Info

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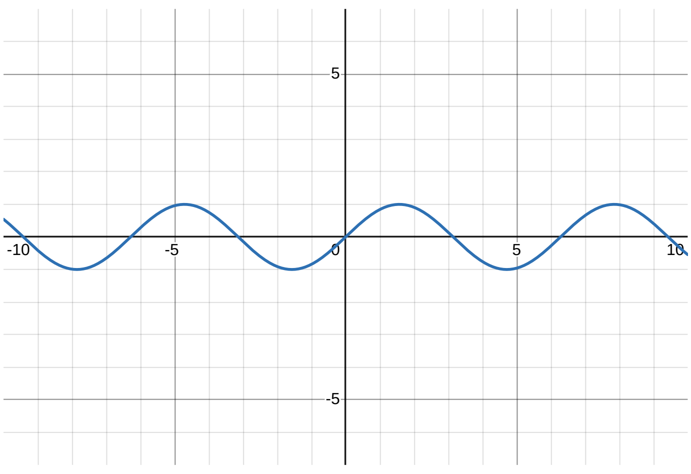
$\sin x$



$\sin x$ is uniformly continuous on $\mathbb{R}$

Let $x, y \in \mathbb{R}$ then

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Now

$$\begin{aligned} |\sin x - \sin y| &\leq \left|2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right| \\ &\leq 2 \end{aligned}$$

For any $\epsilon \geq 0$ choose $\delta := \min\{2, \epsilon\}$. Then

$$|x-y| < \delta \implies |\sin x - \sin y| \leq 2 < \delta \leq \epsilon$$

But it's actually very easy to show this using

> **(Lagrange's mean value theorem)** Let a continuous function
>
> $$f : [a, b] \to \mathbb{R}$$
>
> be differentiable on (a, b). Then there is a point $c \in (a, b)$ at which
>
> $$f(b) - f(a) = (b-a) f'(c)$$
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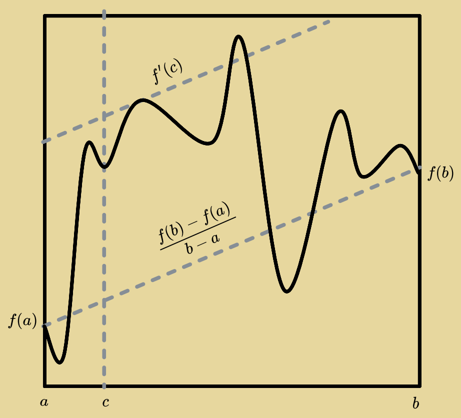
> 
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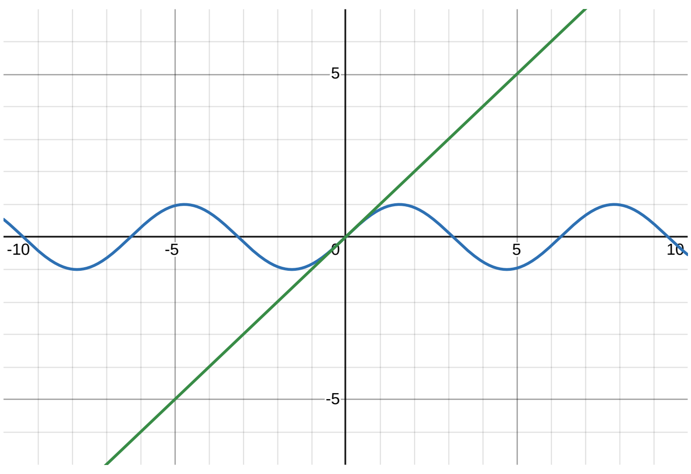
as $\frac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$ we have

$$\begin{aligned} &\forall x, y \in \mathbb{R}, \exists z \in (x, y) \text{ such that} \\ &|\sin x - \sin y| \leq |x-y|\,|\cos z| \\ &\leq |x-y| \end{aligned}$$

So choosing $\delta = \epsilon$ does it.

$|\sin x| \leq |x|$

We may observe



geometrically

invertmix

$$\begin{aligned} &\text{interior of } \triangle ABC \subseteq \text{interior of } \triangle ABD \\ \implies &\text{area}(\triangle ABC) \leq \text{area}(\triangle ABD) \\ \implies &|\sin x| \leq |x| \end{aligned}$$

for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

by the mean value theorem

- As

  $$\sin x : [\epsilon, 1] \to \mathbb{R}$$

  is continuous and differentiable on the interior, by

(Rolle's theorem extended by Cauchy) Let a continuous

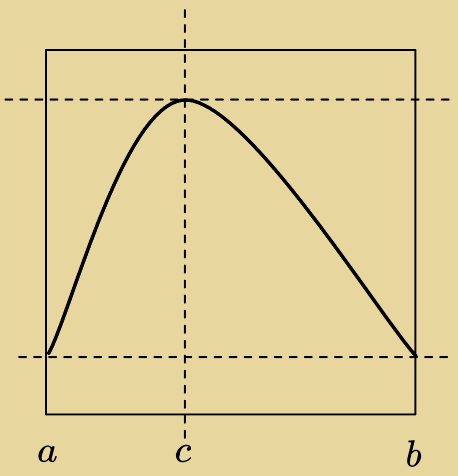
$$f : [a, b] \to \mathbb{R}$$
$$f(a) = f(b)$$

such that

$$f' : (a, b) \to \mathbb{R} \cup \{\infty, -\infty\}$$

exists. Then **there is a point in the interior where the derivative is zero**

$$\exists c \in (a, b) \text{ such that } f'(c) = 0$$



- **(Extremum value theorem for continuous $[a, b] \to \mathbb{R}$)** Let

  $$f : [a, b] \to \mathbb{R}$$

  be **continuous**, then a global maxima *and* a minima exists $M, m \in [a, b]$ for f.

  By

  A continuous function from a compact metric space to $\mathbb{R}$ has a global maxima *and* a global minima.

- Define M, m such that $f(M) = \sup f$ and $f(m) = \inf f$
  - If one of M, m is in the interior (a, b) then $f'(m) = 0$ or $f'(M) = 0$ at that extremum value because of

    (Derivative, if it exists, must be 0 at local extremas) Let a function $f : (a, b) \to \mathbb{R}$ have a local maxima or minima at $c \in (a, b)$. If f has an extended derivative at c then the derivative must be zero

    $$f'(c) \in \mathbb{R} \cup \{-\infty, \infty\} \implies f'(c) = 0$$

    Therefore,

    $$\{\text{local extremum of } f\} \subseteq Z(f')$$

  - If $M, m \in \{a, b\}$, then

    $$f(a) = f(b) \implies M = m$$

    so the minima and maxima values are equal so the function must be a constant: $f(a) = f(b) = f(x)$ for all $x \in (a, b)$. A constant function is always differentiable and derivative is zero: $f'(x) = 0$ for all $x \in (a, b)$.

for every $x \in [0, 1]$ there exists a c such that

$$\frac{\sin(x) - \sin(0)}{x - 0} = \cos(c) \leq 1$$

- Hence, on $[0, 1]$ we have

  $$\sin(x) \leq x$$

- For $x \in [1, \infty]$ we know

  $$\sin x \leq 1$$

  on $\mathbb{R}$, hence

  $$1 \leq x \implies \sin x \leq x$$

Current note has 1 direct children and 1 total descendants.

- squishy
  - pow
    - $\sin x$
      - sin is covering

And it has 28 siblings.

- squishy
  - pow
    - Angle function
    - Bessel functions
    - $$\cos \sqrt{z}$$
    - The complex logarithm
    - $\sum_{n \geq 0} z^{n!}$
    - $z^3 - 3z$
    - $$\frac{r_0}{1 + \epsilon \cos b\theta}$$

- $\cos\left(\pi\frac{p}{q}\right)$
- $\frac{\cos x^k}{x}$
- Elliptic integrals and Jacobi elliptic functions
- The complex exponential $\exp(z)$
- $\exp\left(\frac{z+1}{z-1}\right)$
- e^{-x^2}
- [Only sum terms that are multiples of N in $\mathbb{C}[[z](https://rupadarshiray.github.io/notes/YchnxQ9SF1RsuKjBntMv8a.$
]]
- $\int\frac{1}{x^2+bx+c}$
- $\frac{\exp(z)}{1-z}$
- $\sin x$
- $\sin(x\sin(x))$
- $\frac{\sin x}{x}$
- stock.sin x by xk
- $\sin x^2$
- $\sqrt{z}$
- $\sqrt{z^2-1}$
- Theta functions
- Weierstrass elliptic function $\wp$
- Weierstrass' primary factors
- $x^2\sin\frac{1}{x}$
- $x+x^2\sin\frac{1}{x}$