Info

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Set of all metrics on a set

Fix a set X and let

$$\mathscr{M}_X \subset \mathbb{R}^{X \times X}$$

be the set of all metrics on $\mathscr{M}_X$

Then

$$\mathscr{M}_X \subset [0, \infty)^{\binom{X}{2}}$$

There are following structures on this set

- **evaluation maps**

$$\mathrm{ev}_{x,y} : \mathscr{M}_X \to [0, \infty)$$

  for every $\{x, y\} \in \binom{X}{2}$
- the **topology** induced by a metric

$$\mathsf{Top} : \mathscr{M}_X \to 2^{2^X}$$

  - where two metrics in preimage of a topology are called **equivalent**, whose classes live in

$$\frac{\mathscr{M}_X}{\mathsf{Top}}$$

- $$+$$

  as **sum** of two metrics is a metric
- the **length "metric"** of a metric

$$\begin{aligned} \mathsf{L} : \mathscr{M}_X &\to [0, \infty]^{\binom{X}{2}} \\ d &\mapsto d_{\mathsf{L}} \end{aligned}$$

  - L is identity on the image
  - $$\mathsf{L}(\mathscr{M}_X) \cap \mathscr{M}_X$$

    are called **length metrics**
- the **bounding** of a metric

$$d \mapsto \frac{d}{1 + d}$$

- in general

$$\mathscr{H} := \{f : [0, \infty) \to [0, \infty) \mid f(0) = 0, \text{strictly increasing, concave}\}$$

  "acts" on $\mathscr{M}_X$ by

$$d \overset{f}{\mapsto} f \circ d$$

  - the classes of metrics on X upto a composition with f live in

$$\frac{\mathscr{M}_X}{\mathscr{H}}$$

> Is
>
> $$\mathscr{M}_X \cup -\mathscr{M}_X \cup \{0\}$$
>
> a subspace of
>
> $$\mathbb{R}^{X \times X}$$
>
> ?

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - Metric spaces
    - Set of all metrics on a set

And it has 11 siblings.

- structures based on sets
  - Metric spaces
    - Set of all metrics on a set
    - CAT(0)
    - Space of all compact metric spaces up to isometry
    - Y^X
    - Gromov hyperbolic spaces
    - Metric space isometries
    - Constructing more metrics

- Metric spaces
- Metric simplicial complexes
- Metric subspaces
- Uniformly discrete spaces of bounded geometry