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Zeros and singularities of holomorphic functions

singularities are *not* isolated

under small perturbations

the number of zeroes inside a simple closed contour is invariant under small perturbations

counting zeros and singularities

- Let holomorphic $f : U(\text{open}) \rightarrow \mathbb{C}$ has zeroes and poles in $A \subseteq U$ such that A has no limit points in U . Then $\frac{f'}{f} : U \setminus A \rightarrow \mathbb{C}$ is a holomorphic function.
 - Suppose γ is a null homologous path in U then

$$\frac{1}{2\pi i} \int_{\gamma} \frac{f'}{f} dz = \sum_{a \in A} W(\gamma, a) \text{Res}_a \left(\frac{f'}{f} \right)$$

where the right side is a finite sum as $W(\gamma, a) = 0$ for all but finitely many $a \in A$.

- Near each $a \in A$, $f(z) = (z - a)^k h(z)$ where $h(a) \neq 0$. It follows then

$$\frac{f'(z)}{f(z)} = \frac{k}{z - a} + \underbrace{\frac{h'(z)}{h(z)}}_{\text{holomorphic near } a}$$

- It follows that $k = \text{Res}_a \left(\frac{f'}{f} \right)$.

Definition. Order of a function

Let $\text{ord}_a(f)$ to be the integer k such that $\frac{f(z)}{(z-a)^k}$ extends to a non-vanishing holomorphic function around a . This means k is the order of the zero of f at a , or $-k$ is the order of the pole at a or $f(a)$ is defined and non-zero.

- Then

$$\frac{1}{2\pi i} \int_{\gamma} \frac{f'}{f} dz = \sum_{a \in A} W(\gamma, a) \text{ord}_a(f)$$

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