Info

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Size of holomorphic image of disks

Let

$$\begin{gathered} f : D \to \mathbb{C} \\ f(0) = 0 \end{gathered}$$

be a holomorphic map. Then it has two quantities associated with it

- the *magnitude of its derivative at* 0

$$|f'(0)|$$

- the *sup norm*

$$\|f\|_\infty$$

bounded

If $\|f\|_\infty < \infty$, we may scale f so that one of them is 1. Let us consider this case

$$f \in \mathscr{O}(D), f(0) = 0, f'(0) = 1, \|f\|_\infty < \infty$$

that is, elements of

$$\mathscr{O}^\infty(D) \cap \ker \mathrm{ev}_0$$

with derivative 1, which an affine subspace

$$\mathscr{O}^\infty(D) \cap \ker \mathrm{ev}_0 \cap \mathfrak{D}_0^{-1}(1)$$

These are the bounded functions of the form

$$f(z) = z + a_2 z^2 + \dots$$

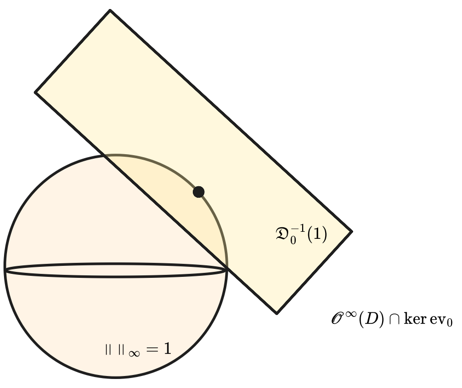
By

> **(Cauchy's bounds on derivatives of holomorphic functions)** Let $f \in \mathscr{O}(U)$. Then
>
> $$\frac{|f^{(n)}(a)|}{n!} \le \frac{\|f|_{B(r,a)}\|_\infty}{r^n}$$

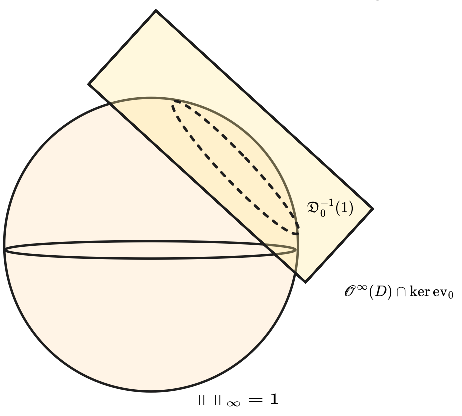
such functions have sup norm at least 1.

> $$\| \ \|_\infty : \mathscr{O}^\infty(D) \cap \ker \mathrm{ev}_0 \cap \mathfrak{D}_0^{-1}(1) \to [1, \infty)$$

The functions with $\| \ \|_\infty = 1$ are precisely rotations.



However, the functions with $\| \ \|_\infty = 1 = \beta > 1$ form a larger space.



> **Definition.** Conformal radius of pointed simply connected open subsets of $\mathbb{C}$
>
> Consider a simply connected open subset $\Omega \subseteq \mathbb{C}$ and some $a \in \Omega$. The unique map in
>
> > **(Riemann mapping theorem)** Let $\Omega \subset \mathbb{C}$ be a simply connected open, proper subset of $\mathbb{C}$. Then there exists a biholomorphism from Ω onto the unit disk
> >
> > $$f : \Omega \to D$$
> >
> > Moreover, given $a \in \Omega$ there is a **unique** biholomorphism
> >
> > $$\begin{gathered} f : \Omega \to D \\ f(a) = 0, f'(a) > 0 \end{gathered}$$
>
> has a $f'(a) \in (0, \infty)$. The **conformal radius** of (Ω, a) is
>
> $$\mathrm{rad}_a(\Omega) := \frac{1}{f'(a)}$$

$\mathscr{O}^\infty(D) \cap \ker \mathrm{ev}_0 \cap \mathfrak{D}_0^{-1}(1) = z + z^2 \mathscr{O}^\infty(D)$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash\mathbb{R}^n, \Gamma\backslash\mathbb{C}^n$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - mapping
        - Size of holomorphic image of disks

And it has 13 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash\mathbb{R}^n, \Gamma\backslash\mathbb{C}^n$
    - Holomorphic functions on spaces over $\mathbb{C}$ of dimension 1
      - mapping
        - Bounding derivative of holomorphic mappings
        - Bounding holomorphic functions linearly
        - Branch points and ramification index of holomorphic functions
        - Disk in holomorphic image of disk
        - Size of holomorphic image of disks
        - Maximum and minimum of holomorphic functions
        - Fibers of holomorphic mappings
        - Non-constant holomorphic functions locally look like w^k and are open mappings
        - Biholomorphic mapping onto disk
        - Conformal radius of pointed simply connected open subsets of $\mathbb{C}$
        - Holomorphic functions on punchered disk
        - Holomorphic function whose singularities are not isolated
        - A holomorphic map branched over $\mathbb{Q}$