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Kaluza-Klein geodesic flow from Lorentz force flow

Definition. Kaluza-Klein geodesic flow from Lorentz force flow

(3') **Kaluza-Klein Theory.** A particle with charge e on a Lorentzian manifold (Q, g) in an electromagnetic vector potential follows a path with

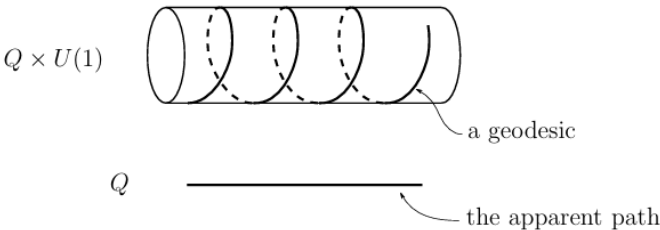
$$\ddot{q}_i = -\Gamma_{ijk}\dot{q}^j\dot{q}^k + \frac{e}{m}F_{ij}\dot{q}^j$$

where

$$F_{ij} = \partial_i A_j - \partial_j A_i$$

but this is actually geodesic motion on the manifold $Q \times U(1)$ where $U(1) = \{e^{i\theta} : \theta \in \mathbb{R}\}$ is a circle.

Let's examine this last result a bit further. To get the desired equations for motion on $Q \times U(1)$ we need to give $Q \times U(1)$ a cleverly designed metric built from g and A where the amount of "spiralling"—the velocity in the $U(1)$ direction is e/m . The metric h on



$Q \times U(1)$ is built from g and A in a very simple way. Let's pick coordinates x^i on Q where $i \in \{0, \dots, n\}$ since we're in $n + 1$ -dimensional spacetime, and θ is our local coordinate on S^1 . The components of h are

$$\begin{aligned} h_{ij} &= g_{ij} + A_i A_j \\ h_{\theta i} &= h_{i\theta} = -A_i \\ h_{\theta\theta} &= 1 \end{aligned}$$



Working out the equations for a geodesic in this metric we get

$$\begin{aligned} \ddot{q}_i &= -\Gamma_{ijk}\dot{q}^j\dot{q}^k + \frac{e}{m}F_{ij}\dot{q}^j \\ \ddot{q}_\theta &= 0, \end{aligned}$$

if $\dot{q}_\theta = e/m$. Since F_{ij} is part of the Christoffel symbols for h .
To summarize this section on least time versus least action we can say that every problem that we've discussed in classical mechanics can be regarded as geodesic motion!

[1]

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