Info

This note, that was found here, as a part of a collection
is written (completely with human hands) by Rupadarshi Ray,
was created on May 22, 2026 3:35:43 PM,
was last modified on September 7, 2026 1:10:49 PM, and
was exported on September 7, 2026 3:09:50 PM.

Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$

Intuition

Let

$$f : U \text{ (open)} \subseteq \mathbb{R}^n \to \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the linear map

$$\mathfrak{D}_p f : T_p\mathbb{R}^n \to T_p\mathbb{R}^m$$

such that

$$f(p+v) \approx f(p) + \mathfrak{D}_p f(v)$$

Definition. Zooming at a point

Let

$$f : U \text{ (open)} \subseteq \mathbb{R}^n \to \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the affine map

$$\mathfrak{Z}_p f : \mathbb{R}^n \to \mathbb{R}^m$$

such that

$$\begin{aligned} & \mathfrak{Z}_p(p+v) && = f(p) + \mathfrak{D}_p f(v) \\ \iff & \mathfrak{Z}_p(x) && = f(p) + \mathfrak{D}_p f(x-p) \end{aligned}$$

where $\mathfrak{D}_p f$ is the derivative at the point

Definition. Derivative of a map $U \subseteq \mathbb{R}^n \to \mathbb{R}^m$ at a point

Let

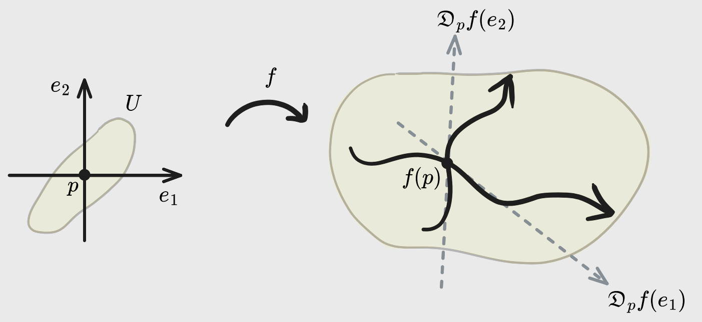
$$f : U \text{ (open)} \subseteq \mathbb{R}^n \to \mathbb{R}^m$$

be a map. Then the **derivative** of f at $p \in U$ is the linear map

$$\mathfrak{D}_p f : T_p\mathbb{R}^n \to T_p\mathbb{R}^m$$

such that

$$\begin{aligned} & \lim_{v\to 0} \frac{f(p+v) - f(p) - \mathfrak{D}_p f(v)}{|v|} = 0 \\ \iff & \lim_{v\to 0} \frac{|f(p+v) - f(p) - \mathfrak{D}_p f(v)|}{|v|} = 0 \end{aligned}$$



$$\begin{aligned} \iff & \lim_{x\to p} \frac{|f(x) - f(p) - \mathfrak{D}_p f(x-p)|}{|x-p|} = 0 \\ \iff & \forall \epsilon > 0 \exists r(\epsilon) > 0 : \\ & \forall x \in B_{r(\epsilon)}(p), |f(x) - f(p) - \mathfrak{D}_p f(x-p)| \leq \epsilon |x-p| \end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash^{\mathbb{R}^n}, \Gamma\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Zooming of a map $\mathbb{R}^n \to \mathbb{R}^m$

And it has 10 siblings.

- stem
  - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma\backslash^{\mathbb{R}^n}, \Gamma\backslash^{\mathbb{C}^n}$
    - Differentiable functions
      - Functions $(a,b) \to \mathbb{R}$ differentiable at a point
      - Comparing a function and it derivative
      - Absolutely continuous functions on $[a,b] \leftrightarrow \int_{[a,-]}(L^1[a,b])$
      - Distributional derivatives
      - Double derivative/Laplace operator on the circle
      - Fractional derivative
      - Infinite limit of derivatives
      - Space of continuous and continuously differentiable functions on $\mathbb{R}$
      - Derivative of maps $\mathbb{R}^n \to \mathbb{R}^m$