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## Tangent bundle $TM$ of a smooth manifold $M$

For any smooth  $n$ -manifold  $M$ ,

$$TM := \bigcup_{p \in M} \{p\} \times T_p M$$

has a natural *topology and smooth structure* to make it a smooth  $2n$ -manifold such that  $TM \rightarrow M$  is smooth vector bundle over  $M$ .

**Definition.** Tangent bundle  $TM \rightarrow M$

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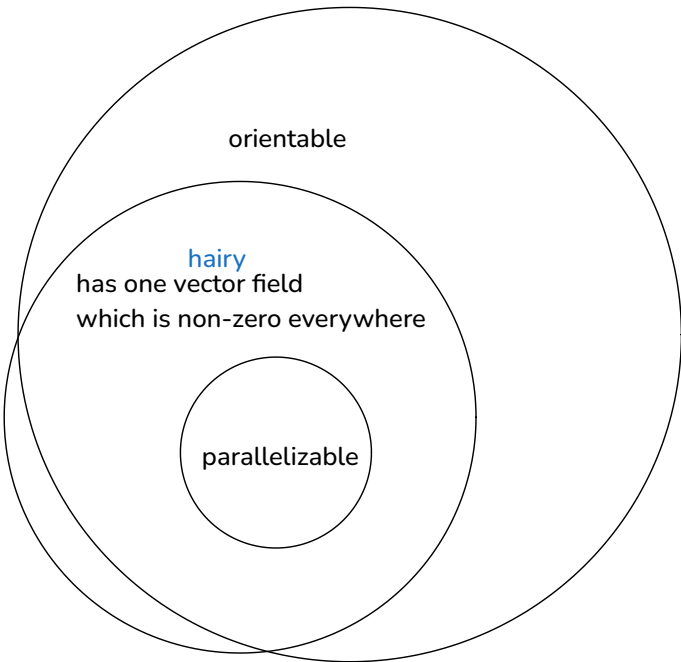
is defined as the **tangent bundle** which is a smooth  $\mathbb{R}^{2 \dim M}$ -vector bundle on a smooth manifold  $M$  with the projection

$$\pi_{TM} : TM \rightarrow M$$

whose fibres are [tangent spaces](#)

$$\pi_{TM}^{-1}(p) = T_p M$$

We represent  $(p, X_p)$  as just  $X_p$ .



## tangent bundle functor

**Definition.** tangent functor

Define the functor

$$T(-) : \mathbf{Man} \rightarrow \mathbf{Vec}_{\mathbb{R}} \mathbf{BunMan}$$

such that

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- $(F : M \rightarrow N) \mapsto (F_* : TM \rightarrow TN)$

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is a functor.

Current note has 0 direct children and 0 total descendants.

- [structures based on sets](#)
  - [manifolds](#)
    - [smooth  \$\mathbb{R}\$ -manifold](#)
      - [Tangents on a smooth manifold](#)
        - [Tangent space of a smooth manifold](#)

- Tangent bundle  $TM$  of a smooth manifold  $M$

And it has 8 siblings.

- structures based on sets
  - manifolds
    - smooth  $\mathbb{R}$ -manifold
      - Tangents on a smooth manifold
        - Tangent space of a smooth manifold
          - Canonical bijection of  $T_pM$  with equivalence classes of velocities
          - Tangents on smooth manifolds as derivations  $\mathcal{C}^\infty(M) \rightarrow \mathbb{R}$
          - local basis of a tangent space of a smooth manifold
          - Tangent bundle  $TM$  of a smooth manifold  $M$
          - Cotangent bundle  $T^*M$  on a smooth manifold  $M$
          - Comparison of constructions with tangent vectors
          - Tangent frames on smooth manifolds
          - Tangent vector fields on a smooth manifold