ⓘ Info

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Rational maps

$$\mathrm{Rat}_d(\mathbb{C}) = \left\{ f \in \mathrm{Hom}\mathsf{Man}_{\mathbb{C}}(\mathbb{C}\boldsymbol{P}^1, \mathbb{C}\boldsymbol{P}^1) \middle| \deg f = d \right\}$$

upto double conjugation

upto conjugation

[1]

upto left composition

When we think of rational functions as morphisms, they are ramified covers of $\boldsymbol{P}^1$ where the domain is also $\boldsymbol{P}^1$

$$\begin{tikzcd} \boldsymbol{P}^1 \arrow[d, "d"] \\ \boldsymbol{P}^2 \end{tikzcd}$$

Then two such covers are isomorphic if there is a isomorphism $\Phi : \boldsymbol{P}^1 \cong_{\mathsf{Man}_{\mathbb{C}}} \boldsymbol{P}^1$ such that

$$\begin{tikzcd} \boldsymbol{P}^1 \arrow[r, "\Phi"] & \boldsymbol{P}^1 \arrow[d, "d"] \\ & \boldsymbol{P}^2 \end{tikzcd}$$

commuties.

This is equivalent to thinking of such maps as endomorphisms of $\boldsymbol{P}^1$ and letting the automorphism group act on it

$$\begin{gathered} \mathrm{Aut}\mathsf{Man}_{\mathbb{C}}(\boldsymbol{P}^1) \curvearrowright \mathrm{End}\mathsf{Man}_{\mathbb{C}}(\boldsymbol{P}^1) \\ f \mapsto \Phi \circ f \end{gathered}$$

Current note has 0 direct children and 0 total descendants.

- stem
    - Objects stemming from $\mathbb{R}$-spaces of dimension 2
        - Riemann surfaces
            - Rational maps

And it has 16 siblings.

- stem
    - Objects stemming from $\mathbb{R}$-spaces of dimension 2
        - Riemann surfaces
            - Vector bundles on complex 1-manifolds
            - Classification of complex 1-manifold
            - Riemann's existence of holomorphic maps between compact Riemann surfaces
            - Divisors on compact Riemann surfaces
            - Fluid
            - Meromorphic differentials on Riemann surfaces
            - Holomorphic functions on a complex 1-manifold
            - Holomorphic maps of type $0 \xrightarrow{3} 0$
            - Rational maps
            - Hurwitz scheme
            - Moduli space of $\mathbb{C}$-manifolds
            - Meromorphic functions on a Riemann surface
            - Normalization of plane projective $\mathbb{C}$-curves and Riemann surfaces
            - Riemann surfaces defined over a number field
            - Hyperelliptic Riemann surfaces
            - Riemann surfaces with a holomorphic 1-form

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1. https://math.hawaii.edu/~mmanes/Talks/Mdtalk.pdf ↩