> ⓘ Info
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Tangent bundle of a smooth vector bundle

Definition. Tangent bundle of a smooth vector bundle and its vertical sub-bundle

Let

$$\begin{array}{c} (E, +_E, \ _E) \\ \big\downarrow{\scriptstyle \pi_E} \\ M \end{array}$$

be a smooth vector bundle over a smooth manifold M.

Then we have the *tangent bundle* of the vector bundle

$$\begin{array}{c} TE \\ \big\downarrow{\scriptstyle \pi_{TE}} \\ E \end{array}$$

and a commutative diagram

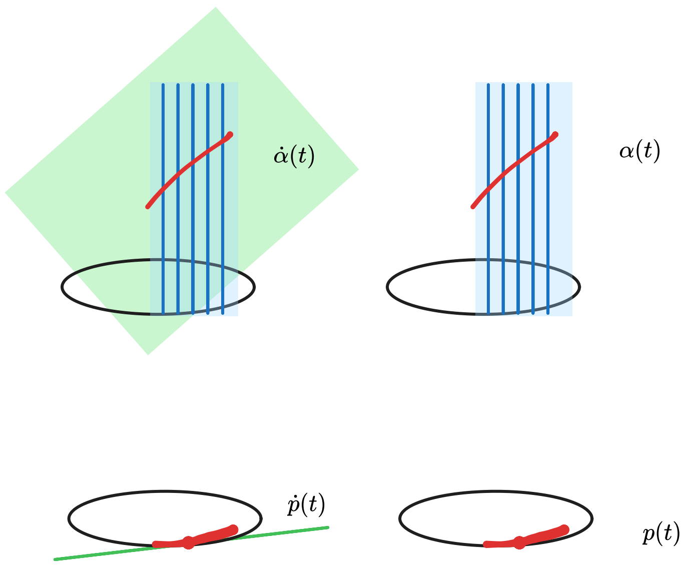
$$\begin{tikzcd} TE \arrow[r, "\pi_{TE}"] \arrow[d, "\mathfrak{D}\pi_E"] & E \arrow[d, "\pi_E"] \\ TM \arrow[r, "\pi_{TM}"] & M \end{tikzcd}$$

Then

$$\ker(\mathfrak{D}\pi_E) \leq TE$$

is the **vertical sub-bundle**.

smooth manifold/vector bundle	atlas
$TM \xrightarrow{\pi_{TM}} M$	Let $(x_\alpha : U_\alpha \to \mathbb{R}^{\dim M})_{\alpha \in A}$ be a **smooth atlas** for M.
	Then $(\mathfrak{D}x_\alpha : TU_\alpha \to \mathbb{R}^{\dim M} \times \mathbb{R}^{\dim M})_{\alpha \in A}$ be the **vector bundle atlas** for TM
$\begin{array}{c} E \\ \big\downarrow{\scriptstyle \pi_E} \\ M \end{array}$	Let $(\psi_\alpha : E_{U_\alpha} \to U_\alpha \times \mathbb{R}^k)_{\alpha \in A}$ be a vector bundle atlas for E.
E	Then $(\psi'_\alpha := (x_\alpha \times \mathrm{Id}_{\mathbb{R}^n}) \circ \psi_\alpha : E_{U_\alpha} \to x_\alpha(U_\alpha) \times \mathbb{R}$ is a smooth atlas for E, giving the coordinates $(x_\alpha^1, \ldots, x_\alpha^{\dim M}, \xi_1, \ldots, \xi_k)$ on E_{U_α}.
$TE \xrightarrow{\pi_{TE}} E$	$\left(\mathfrak{D}\psi_\alpha : TE_{U_\alpha} \to \begin{matrix} x(U_\alpha) \times \mathbb{R}^k \\ \times \mathbb{R}^{\dim M + k} \end{matrix}\right)_{\alpha \in A}$ is the vector bundle atlas for TE
TE	We have the coordinates $\begin{pmatrix} x_\alpha^1, \ldots, x_\alpha^{\dim M}, \xi_1, \ldots, \xi_k \\ \eta_1, \ldots, \eta_{\dim M}, \zeta_1, \ldots, \zeta_k \end{pmatrix}$ on TE_{U_α}
$\begin{array}{ccc} TE_{U_\alpha} & \xrightarrow{\pi_{TE}} & E_{U_\alpha} \\ & & \big\downarrow{\scriptstyle \pi_E} \\ TU_\alpha & \xrightarrow{\pi_{TM}} & U_\alpha \end{array}$	The diagram looks like $\begin{pmatrix} x_\alpha^1, \ldots, x_\alpha^{\dim M}, \xi_1, \ldots, \xi_k \\ \eta_1, \ldots, \eta_{\dim M}, \zeta_1, \ldots, \zeta_k \end{pmatrix} \mapsto (x_\alpha^1, \ldots, x$ $\begin{pmatrix} x_\alpha^1, \ldots, x_\alpha^{\dim M} \\ \eta_1, \ldots, \eta_{\dim M} \end{pmatrix} \mapsto (x_\alpha^1, \ldots$
$\begin{array}{c} TE \\ \big\downarrow{\scriptstyle \mathfrak{D}\pi_E} \\ TM \end{array}$	
$VE \leq TE$	$\begin{pmatrix} x_\alpha^1, \ldots, x_\alpha^{\dim M}, \xi_1, \ldots, \xi_k \\ 0, \ldots, 0, \zeta_1, \ldots, \zeta_k \end{pmatrix}$



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And it has 5 siblings.

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