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## Geodesic completeness $\iff$ metric completeness of Riemannian manifolds

**(Hopf-Rinow)** Let  $(M, g)$  be a connected Riemannian manifold. Then  $M$  is geodesically complete

$$\iff$$

$(M, d_g)$  is a complete metric space

$$\iff$$

$\exp_p : T_p M \rightarrow M$  is defined for some  $p \in M$

$$\iff$$

every closed  $d$ -bounded set in  $M$  is compact.  
If  $(M, g)$  satisfies any (hence all) of the above, then each pair of points in  $M$  can be joined by a **length-minimizing** geodesic segment. In particular, for each  $p \in M$  the exponential

$$\exp_p : T_p M \rightarrow M$$

is surjective.

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- [structures based on sets](#)
  - [manifolds](#)
    - [ℝ-manifold semi-Riemannian](#)
      - [Complete Riemannian manifolds](#)
        - [Geodesic completeness](#)  $\iff$  [metric completeness of Riemannian manifolds](#)

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- [structures based on sets](#)
  - [manifolds](#)
    - [ℝ-manifold semi-Riemannian](#)
      - [Complete Riemannian manifolds](#)
        - [Integrating sectional curvature](#)
        - [Riemannian manifolds with negative curvature](#)
        - [Riemannian manifolds with non-positive sectional curvature](#)
        - [Cut locus in a complete Riemannian manifold](#)
        - [Exponential map for a complete Riemannian manifold](#)
        - [Geodesic completeness](#)  $\iff$  [metric completeness of Riemannian manifolds](#)
        - [Geodesics in homotopy classes](#)
        - [Homogeneous Riemannian manifolds](#)
        - [Homogeneous and isotropic Riemannian manifolds](#)
        - [Poles of a complete Riemannian manifold](#)
        - [Riemannian symmetric spaces](#)
        - [Uniquely geodesic complete Riemannian manifolds](#)
        - [Riemannian homogeneous manifolds where  \$\exp\_p\$  is a diffeomorphism](#)
        - [Complete Riemannian manifolds without conjugate points](#)