Info

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$H(3,\mathbb{R}) = N(3,\mathbb{R})$

#Wiki/group/Lie

Definition.

$$H(3,\mathbb{R}) := \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \middle| a,b,c \in \mathbb{R} \right\}$$

$$\begin{bmatrix} 1 & a_1 & b_1 \\ 0 & 1 & c_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a_2 & b_2 \\ 0 & 1 & c_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & a_1+a_2 & b_1+b_2+a_1c_2 \\ 0 & 1 & c_1+c_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Proposition:

$$H(3,\mathbb{R}) \cong \left(\begin{matrix} \mathbb{R}^3 \\ \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} a_1+a_2 \\ b_1+b_2+a_1c_2 \\ c_1+c_2 \end{bmatrix} \end{matrix} \right)$$

Proposition:

$$\begin{aligned} Z(H(3,\mathbb{C})) = \left\{ \begin{bmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \middle| b \in \mathbb{R} \right\} &\cong \mathbb{R} \\ \begin{bmatrix} 1 & 0 & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} &\mapsto b \end{aligned}$$

Proposition:

$$\begin{aligned} \mathfrak{n}(3,\mathbb{R}) = \mathfrak{h}(3,\mathbb{R}) &\cong \mathbb{R}\langle I, X, Y | [X,Y] = I \rangle \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} &\mapsto X \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} &\mapsto Y \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} &\mapsto I \\ \begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{bmatrix} &\mapsto aX + bI + cY \end{aligned}$$

Definition.

$$\begin{aligned} \mathfrak{h}(3,\mathbb{R}) &\to \mathfrak{gl}(\mathcal{C}^\infty(\mathbb{R})) \\ f &\overset{X}{\mapsto} (\pi i) x f(x) \\ f &\overset{Y}{\mapsto} f' \\ f &\overset{I}{\mapsto} -\pi i \mathrm{Id} \end{aligned}$$

$$aX + bI + cI \mapsto a\pi i\tilde{x} + b\mathfrak{D} - c\pi i\mathrm{Id}$$

$$\rho_\lambda : H(3,\mathbb{R}) \curvearrowright \mathcal{B}(L^2(\mathbb{R}))$$

Proposition: Any normal subgroup of $H(3,\mathbb{R})$ either contains the center $Z(H(3,\mathbb{R}))$ or is contained in it.

Current note has 1 direct children and 1 total descendants.

- squishy
  - Heisenberg group
    - $H(3,\mathbb{R}) = N(3,\mathbb{R})$
      - $H(3,\mathbb{R})/\mathbb{Z}$

And it has 2 siblings.

- squishy
  - Heisenberg group
    - $H(3,\mathbb{C})$
    - $H(3,\mathbb{R}) = N(3,\mathbb{R})$