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Basis of functions on $[0, 1], S^1$

Proposition: The $\mathbb{C}$ -span of the following are dense in the corresponding spaces

- $\{x^n|n \geq 1\} \subseteq L^2[0, 1]$
- $\{z^n|n \in \mathbb{Z}\} \subseteq L^2(S^1)$
- $\{1\} \cup \{\sin(2n\pi x)|n \geq 1\} \cup \{\cos(2n\pi x)|n \geq 1\}$
- $\{\sin(n\pi x)|n \geq 1\}$
- $\{\cos(n\pi x)|n \geq 1\}$

Interestingly, this implies, *even* functions in $L^2[0, 1]$ are dense! Similarly, odd functions.

Current note has 0 direct children and 0 total descendants.

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 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
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And it has 2 siblings.

- [stem](#)
 - [subobjects of and functions on \$\mathbb{R}^n, S^n, \mathbb{C}^n\$ and its quotients \$\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n\$](#)
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