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Asymptotics of monotonically increasing functions

$[0, \infty] \rightarrow [0, \infty]$

Let

$$f, g : [0, \infty] \rightarrow [0, \infty]$$

be functions.

Then	if	(also equivalent to)
$f \lesssim g$ or $g \gtrsim f$ or $f = O(g)$	f is less that some positive scaling of g $\exists c > 0 : f \leq cg$ "f grows at worst g-ly"	
"eventually $f \lesssim g$ "		
f is comparable to g $f \asymp g$	f can be sandwiched between positive scalings of g $\exists c_1, c_2 > 0 : c_1g \leq f \leq c_2g$	$f \lesssim g$ and $g \lesssim f$
f is negligible to g $f \ll g$ or $f = o(g)$	$\lim_{t \rightarrow \infty} \frac{f(t)}{g(t)} = 0$ $\iff \forall \epsilon > 0 \exists T > 0 : f \leq \epsilon g$ on (T, ∞) $\iff \forall \epsilon > 0, f \lesssim \epsilon g$ on (T_ϵ, ∞)	
$\rho_1 \preceq \rho_2$	ρ_1 is less than some positive scaling and translate of a scaled and translated ρ_2 $\exists a, b > 0, c, d \geq 0 : \rho_1(r) \leq a\rho_2(c r + d)$	
$\rho_1 \simeq \rho_2$		$\rho_1 \preceq \rho_2$ and $\rho_2 \preceq \rho_1$
$f \overset{t_0}{\sim} g$ for $t_0 \in [0, \infty]$	$\lim_{t \rightarrow t_0} \frac{f(t)}{g(t)} = 1$	

Exercise

$$r^d \simeq r^D \implies d = D$$

Exercise

$$e^{\alpha r} \simeq e^{\beta r} \text{ for all } \alpha, \beta > 0$$

Current note has 0 direct children and 0 total descendants.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Monotone functions on $\mathbb{R}$
 - Asymptotics of monotonically increasing functions $[0, \infty] \rightarrow [0, \infty]$

And it has 1 siblings.

- stem
 - subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash \mathbb{R}^n, \Gamma \backslash \mathbb{C}^n$
 - Monotone functions on $\mathbb{R}$
 - Asymptotics of monotonically increasing functions $[0, \infty] \rightarrow [0, \infty]$