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Holomorphic tangent bundle of a complex manifold

$$T^{\mathbb{C}}M$$

Definition. The complex structure on $T^{\mathbb{C}}M$ of a complex manifold M

$$J|_{U_i}: T^{\mathbb{R}}U_i \rightarrow T^{\mathbb{R}}U_i$$

$$J^2 = -\text{Id}_{T^{\mathbb{R}}M}$$

In a complex manifold

$$\begin{aligned} T^{\mathbb{C}}M &\rightarrow T^{\mathbb{R}}M \otimes \mathbb{C} \\ \frac{\partial}{\partial z_j} &\mapsto \frac{1}{2} \left(\frac{\partial}{\partial x_j} - i \frac{\partial}{\partial y_j} \right) \end{aligned}$$

is an inclusion of $\mathbb{C}$ -vector bundles.

Definition. Definition

The subbundles

$$T^{1,0}M, T^{0,1}M \leq T^{\mathbb{R}}M \otimes \mathbb{C}$$

such that J has eigenvalues $i, -i$ respectively.

$$T^{\mathbb{C}}M \cong T^{1,0}M \oplus T^{0,1}M$$

for almost complex manifolds

(M, J) , we have

$$T^{\mathbb{R}}M \otimes \mathbb{C}$$

Current note has 0 direct children and 0 total descendants.

- structures based on sets
 - manifolds
 - $\mathbb{C}$ -manifold
 - Holomorphic tangent space of a complex manifold
 - Holomorphic tangent bundle of a complex manifold

And it has 1 siblings.

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