Info

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Bi-invariant metrics on Lie groups

Let G be a Lie group $\mathfrak{g}$ its Lie algebra. Then G admits a bi-invariant metric

$$\iff$$

there exists an $\mathrm{Ad}(G)$-invariant inner product on $\mathfrak{g}$

$$\iff$$

the image $\mathrm{Ad}(G)$ has compact closure in $GL(\mathfrak{g})$.

Let $\mathfrak{g}$ be a Lie algebra with an Ad-invariant inner product. Then $\mathfrak{g}$ is a direct orthogonal sum of simple ideals

$$\mathfrak{g} \cong \underbrace{\mathbb{R}^n}_{\text{Abelian}} \oplus \underbrace{\mathfrak{g}_1 \oplus \cdots \oplus \mathfrak{g}_n}_{\text{semisimple}}$$

where $\mathfrak{g}_i$ are simple non-commutative ideals. Now let G be the *simply connected* Lie group with Lie algebra isomorphic to $\mathfrak{g}$ then

$$G \cong \underbrace{\mathbb{R}^n}_{\text{Abelian}} \times \underbrace{G_1 \times \cdots \times G_n}_{\text{compact}}$$

where G_i are compact Lie groups.

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold group (Lie groups)
      - Bi-invariant metrics on Lie groups

And it has 18 siblings.

- structures based on sets
  - manifolds
    - $\mathbb{R}$-manifold group (Lie groups)
      - Adjoint representation of a Lie group
      - Conjugation action of a Lie group on itself
      - Classification of Lie groups
      - Compact Lie groups
      - Connected Lie groups with discrete center
      - Connected Lie groups with non-surjective exponential map
      - Discrete subgroups of Lie groups
      - Smooth maps onto a Lie group
      - Lie groups with surjective exponential map
      - Maurer-Cartan form
      - Left-invariant Lagrangian on a Lie group
      - Lie algebra of a lie group
      - Bi-invariant metrics on Lie groups
      - Left invariant Riemannian metric on a Lie group
      - Semi-simple $\mathbb{R}$-Lie groups
      - Almost simple groups
      - Virtual subgroups of Lie groups
      - Lie group with dilations