Info

This note, that was found here, as a part of a collection
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Dot product 2-norm on $\mathbb{R}^n$

For any function $f : \mathbb{R}^n \to \mathbb{R}^n$ such that

$$\|x - y\| = \|f(x) - f(y)\|$$

then $f(x) - f(0)$ is an orthogonal map.

$$\begin{aligned}
& \|x-y\| && = \|fx - fy\| \\
& && = \|(fx - f0) - (fy - f0)\| \\
\implies & \langle x-y, x-y\rangle && = \langle (fx-f0)-(fy-f0), (fx-f0)-(fy-f0)\rangle \\
\implies & \underbrace{\langle x,x\rangle}_{\|x-0\|^2} - 2\langle x,y\rangle + \underbrace{\langle y,y\rangle}_{\|y-0\|^2} && = \underbrace{\langle fx-f0, fx-f0\rangle}_{\|fx-f0\|^2} \\
& && -2\langle fx-f0, fy-f0\rangle \\
& && +\underbrace{\langle fy-f0, fy-f0\rangle}_{\|fy-f0\|^2} \\
\implies & \langle x,y\rangle && = \langle fx-f0, fy-f0\rangle
\end{aligned}$$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - $\mathbb{R}^n$
    - Dot product 2-norm on $\mathbb{R}^n$

And it has 4 siblings.

- stretchy
  - $\mathbb{R}^n$
    - Automorphisms of $\mathbb{R}^n$
    - $GL(n, \mathbb{R}^n) \curvearrowright \mathbb{R}^n$
    - Isometric embedding of a subset of $\mathbb{R}^n$ into $\mathbb{R}^n$
    - Dot product 2-norm on $\mathbb{R}^n$