Info

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Integration of measurable functions with respect to a $[0,\infty]$-measure

Definition. Integration of measurable functions with respect to a $[0,\infty]$-measure

Let $A_1,\dots,A_n \in \mathfrak{M}_X$ and $\alpha_1,\dots,\alpha_n \in [0,\infty]$ then the *simple function*

$$\sum_{i=1}^{n} \alpha_i \chi_{A_i} : X \to [0,\infty]$$

has integral is the element of $[0,\infty]$

$$\int_S \left(\sum_{i=1}^{n} \alpha_i \chi_{A_i}\right) \mathrm{d}\mu := \sum_{\substack{1\le i\le n \\ \alpha_i \neq 0}} \alpha_i \mu(A_i \cap E)$$

on some $E \in \mathfrak{M}$.
Let $f : X \to [0,\infty]$ be measurable and $E \in \mathfrak{M}$, then the μ-**integral** of f is

$$\int_E f \mathrm{d}\mu := \sup_{\substack{\text{simple } s: \\ 0\le s\le f}} \int_E s \mathrm{d}\mu$$

an element of $[0,\infty]$.
Let

$$\mathscr{L}^1(X,\mu) := \left\{ f : X \to \mathbb{C} \text{ measurable} : \int_X |f| \mathrm{d}\mu < \infty \right\}$$

then for $f \in \mathscr{L}^1(X,\mu)$

$$\int_E f := \int_E \Re f + i \int_E \Im f$$

L^p spaces

$$L^1(X,\mu) := \frac{\mathscr{L}^1(X,\mu)}{f \sim g \iff f = g \ \mu\text{-ae}}$$

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- structures based on sets
  - Measure spaces
    - $[0,\infty]$-measure spaces
      - Integration of measurable functions with respect to a $[0,\infty]$-measure

And it has 4 siblings.

- structures based on sets
  - Measure spaces
    - $[0,\infty]$-measure spaces
      - Measurable action of a topological group on a positive measure space
      - Integral inequalities
      - Integration of measurable functions with respect to a $[0,\infty]$-measure
      - Monotonic subsets in a measure space