Info

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Convex sets in $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n

Definition. Convex sets in $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n

A subset $C \subseteq \mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or $S^n, n > 0$ is **convex** if for every pair of points $x, y \in C$ such that $d(x, y) < \operatorname{diam}(\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n \text{ or } S^n)$, the (unique) geodesic segment between x, y is contained in C

$$\overline{xy} \subseteq C$$

We have

- the **dimension** of C

$$\dim C := \min\{m \in \mathbb{N} | C \text{ is contained in an } m\text{-plane}\}$$

- $\operatorname{plane}(C)$ is the unique $\dim C$-plane that contains C
- a **side** of C is a non-empty, maximal convex subset of ∂C.

$$\operatorname{sides}(C) := \max(\{\emptyset \neq A \subseteq \partial C | A \text{ is convex}\}, \subseteq)$$

  is the (possibly uncountable) set of all sides of C

$$S \in \operatorname{sides}(C) \implies \overline{C} \cap \operatorname{span}(S) = S$$

[1]

convex polyhedron

Definition. Convex polyhedron

A **convex polyhedron** P in $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n is a non-empty, closed, (geodesically) convex subset of $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n such that the collection $\operatorname{sides}(P)$ of sides of P is locally finite.

Given

> **Definition.** Convex polyhedron
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> A **convex polyhedron** P in $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n is a non-empty, closed, (geodesically) convex subset of $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n such that the collection $\operatorname{sides}(P)$ of sides of P is locally finite.

Then for each side $S \in \operatorname{sides}(P)$ we have

$$\dim S = \dim P - 1$$

Let $P \subseteq \mathbb{R}\boldsymbol{H}^n$ be a convex polyhedron that is a proper subset of $\operatorname{span}(P)$. Then

$$P = \bigcap_{S \in \operatorname{sides}(P)} H_S$$

where H_S is the closed half-space of $\operatorname{span}(P)$ such that $\partial H_S = \operatorname{span}(S)$.

Consider a side $S \in \operatorname{sides}(P)$ of a convex polyhedron $P \in X$. Then each point in the boundary $x \in \partial S$ is in the boundary of another side of P.

Every side of a convex polyhedron P is a convex polyhedron.

geometrically finite convex polyhedron

Definition. Geometrically finite convex polyhedron

A convex polyhedron $P \subseteq \mathbb{R}\boldsymbol{H}^n$ is **geometrically finite** if for each $x \in \partial_\infty P$ there is a onb $N_x \subseteq \overline{\mathbb{R}\boldsymbol{H}^n}$ that meets just the sides of P incident with x

$$\forall H \in \operatorname{sides}(P), N_x \cap H \neq \emptyset \iff x \in \partial_\infty H$$

- finite-sided convex polyhedra $\subset$ geometrically finite convex polyhedra

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Current note has 2 direct children and 2 total descendants.

- stem
  - Convex sets in $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n
    - Polyhedron gluings
    - κ-simplicial complex

And it has 13 siblings.

- stem
  - $GL(n)$
  - $GL(n)(\mathbb{C})$
  - $GL(n)(\mathbb{R})$
  - Knots

- Objects stemming from spaces over $\mathbb{Q}$
- Objects stemming from $\mathbb{R}$-spaces of dimension 2
- Objects stemming from $\mathbb{R}$-spaces of dimension 3
- Objects stemming from $\mathbb{R}$-spaces of dimension 4
- subobjects of and functions on $\mathbb{R}^n, S^n, \mathbb{C}^n$ and its quotients $\Gamma \backslash^{\mathbb{R}^n}, \Gamma \backslash^{\mathbb{C}^n}$
- Convex sets in $\mathbb{R}^n, \mathbb{R}\boldsymbol{H}^n$ or S^n
- Objects stemming from $\mathbb{R}$-semi-simple/reductive Lie groups
- Objects stemming from $PSL(2)(\mathbb{C})$
- Objects stemming from $PSL(2)(\mathbb{R})$

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1. John G. Ratcliffe - Foundations of Hyperbolic Manifolds (2019), p.193 ↩