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Metric spaces

Definition. Metric space (M, d)

A **metric space** (M, d) is a pair consisting of a set M and *distance metric* on M

$$d : M \times M \rightarrow \mathbb{R}$$

such that

- (*non-degeneracy*) $d(x, y) = 0 \iff x = y$
- $d(x, y) = d(y, x)$
- (*triangle inequality*) $d(x, z) + d(z, y) \geq d(x, y)$

Thus, the distance metric is a function

$$d : \mathbf{Sym}^2(M) \rightarrow \mathbb{R}_{>0}$$

that satisfies the triangle inequality.

Let (M, d) be a metric space. Then

$$d(x, y) \geq 0$$

- $d(x, y) + d(y, z) \geq d(z, x)$
- $d(x, y) + d(z, x) \geq d(y, z)$

Then adding the two equations we get

$$2d(x, y) \geq 0 \implies d(x, y) \geq 0$$