Info

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$SL(2)(\mathbb{C})$

#wiki/Man/C

The group $SL(2)(\mathbb{C})$ contains complex matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

such that

$$ad - bc = 1$$

> **Proposition:**
>
> $$SU(2,\mathbb{C}) \times P \to SL(2)(\mathbb{C})$$

- The 3 $\mathbb{C}$-dimensional Lie group $SL(2)(\mathbb{C})$ is diffeomorphic to $S^3 \times \mathbb{R}^3$, this is connected and non-compact.
- The Lie algebra $\mathfrak{sl}(2,\mathbb{C})$ is the set of all traceless 2x2 matrices, spanned by

$$H := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} (\text{traceless diagonal})$$
$$X := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} (= E_{12})$$
$$Y := \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} (= E_{21})$$

- The exponential looks like

$$\exp$$

$$\begin{bmatrix} a & b \\ 0 & -a \end{bmatrix} \mapsto \begin{bmatrix} e^a & \end{bmatrix}$$

- Subgroups
  - upper triangular

$$\begin{bmatrix} a & b \\ & d \end{bmatrix}$$

> **Proposition:** $Z(SL(2)(\mathbb{C})) = \{I, -I\}$

Thus,

$$\begin{array}{c} SL(2)(\mathbb{C}) \\ \downarrow \\ PSL(2)(\mathbb{C}) \end{array}$$

representations

- on $\mathbb{C}^2$
- on $\boldsymbol{Sym}^k(\mathbb{C}^2)$
- on $\mathbb{C}[X,Y]$
- $SL_{\mathbb{C}}(2) \curvearrowright \mathbb{C}(x,y)$
-

conjugacy classes

Let

$$\lambda, \frac{1}{\lambda} = \frac{\mathrm{tr}}{2} \pm \frac{\sqrt{\mathrm{tr}^2 - 4}}{2}$$

be the eigenvalues of $A \in SL(2)(\mathbb{C})$.

$\mathrm{tr}^2 - 4$	$(-\infty, 0)$	0	$(0, \infty)$	
$PSL(2)(\mathbb{C}) \curvearrowright \mathbb{C}P$	fixes one point			