Info

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$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

powers

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{2} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{3} = \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{4} = \begin{bmatrix} 34 & 21 \\ 21 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{5} = \begin{bmatrix} 89 & 55 \\ 55 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{6} = \begin{bmatrix} 233 & 144 \\ 144 & 89 \end{bmatrix}$$

The elements of the powers are elements of the Fibonacci sequence

Definition. Fibonacci sequence

The Fibonacci sequence is defined by the recurrence relation

$$\begin{aligned} F(0) &= 0 \\ F(1) &= 1 \\ F(n+2) &= F(n) + F(n+1) \end{aligned}$$

which looks like

$$0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{20} = \begin{bmatrix} 165580141 & 102334155 \\ 102334155 & 63245986 \end{bmatrix}$$

A square root in $GL(2)(\mathbb{Z})$ is

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{2} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

and because

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n} = \begin{bmatrix} F(n+1) & F(n) \\ F(n) & F(n-1) \end{bmatrix}$$

for all $n \geq 0$ where $F(n)$ is the Fibonacci sequence

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we conclude

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{n} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{2n} = \begin{bmatrix} F(2n+1) & F(2n) \\ F(2n) & F(2n-1) \end{bmatrix}$$

action on $\mathbb{R}^2$: invariant space decomposition

$$\begin{aligned} \det \begin{bmatrix} X-2 & -1 \\ -1 & X-1 \end{bmatrix} &= 0 \\ (X-2)(X-1) - 1 &= 0 \\ X^2 - 3X - 1 &= 0 \end{aligned}$$

That means the

- eigenvalues are

$$\lambda_1, \lambda_2 = \frac{3 \pm \sqrt{5}}{2} = 1 + \underbrace{\frac{1 \pm \sqrt{5}}{2}}_{\varphi, \bar{\varphi}}$$

  whose product $\lambda_1 \lambda_2 = \det = 1$
- and the eigenvectors are

$$\begin{bmatrix} \varphi \\ 1 \end{bmatrix}, \begin{bmatrix} \bar{\varphi} \\ 1 \end{bmatrix}$$

Current note has 0 direct children and 0 total descendants.

- squishy
  - Matrices
    - $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$
- stretchy
  - $GL(2)(\mathbb{Z}) \curvearrowright T^2$
    - Arnold cat map $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \curvearrowright T^2$

And it has 4 siblings.

- squishy
  - Matrices
    - $\begin{bmatrix} 2 & \\ & \frac{1}{2} \end{bmatrix}$
    - $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
    - $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
    - $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$