Info

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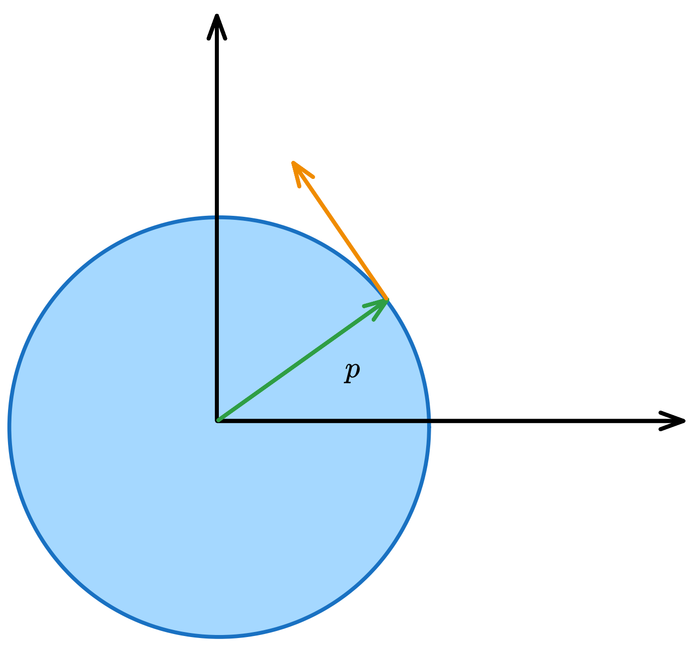
Almost complex structure on S^2 induced from spherical metric and its area form

Definition. Almost complex structure on S^2 using cross product on $\mathbb{R}^3$.

The involution

$$\begin{aligned} J : TS^2 &\to TS^2 \\ X_p &\mapsto p \times X_p \end{aligned}$$



where we've identified $T_pS^2 \leq \mathbb{R}^3$.

Because area spanned by the parallelogram is

$$\|X\| \, \|Y\| \sin(\theta_{X,Y})$$

and *oriented area* is

- supposed to be

$$\begin{aligned} \langle X, Y \rangle &= \|X\| \, \|Y\| \cos(\theta_{X,Y}) \\ &= \|X\| \, \|Y\| \sin\left(\theta_{X,Y} + \frac{\pi}{2}\right) \\ &= \langle JX, Y \rangle \end{aligned}$$

- $$\langle JX, Y \rangle = \langle p \times X, Y \rangle = \langle p, X \times Y \rangle$$

  which, again, *should* be the oriented area of $X, Y \in T_pS^2$
- In local coordinates

$$\langle JX, Y \rangle = \left\langle J\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\rangle \mathrm{d}x \wedge \mathrm{d}y = \left\| \frac{\partial}{\partial x} \times \frac{\partial}{\partial y} \right\| \mathrm{d}x \wedge \mathrm{d}y$$

so we conclude

$$g_{\mathsf{S}}(JX, Y) = \lambda_{\mathsf{S}}(X, Y)$$

Current note has 0 direct children and 0 total descendants.

- stretchy
  - S^2 and $\mathbb{C}\boldsymbol{P}^1$
    - Almost complex structure on S^2 induced from spherical metric and its area form

And it has 10 siblings.

- stretchy
  - S^2 and $\mathbb{C}\boldsymbol{P}^1$
    - Almost complex structure on S^2 induced from spherical metric and its area form
    - Borsuk-Ulam theorem
    - Cylindrical coordinates on S^2
    - Holomorphic vector bundles on $\mathbb{C}\boldsymbol{P}^1$
    - Round metric on S^2
    - The area form on round S^2
    - Stereographic coordinates on the sphere S^2
    - TS^2
    - Vector fields on S^2
    - Hamiltonian flows on the sphere S^2