Info

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Interpretation of measure preserving endomorphisms

X	$x \in X$	
$B \subseteq X$	$x \in B$	
χ_B	$\chi_B(x)$	$\mathtt{check}(x \in B) \in \{0,1\}$
μ	$\mu(B)$	$\mu(B) = \mu\text{–}\mathtt{P}(x \in B)$ is the **size** of B with respect to μ is the **probability** of randomly picking a point x in B
$T : X \to X$ that is μ-invariant	Definition. Measure preserving endomorphism Let $(X, \mathfrak{M}, \mu)$ be a $[0, \infty]$-measure space and $T : X \to X$ be a measurable map. Then T *preserves the measure* μ that is μ is a T**-invariant measure** or T **is** μ**-preserving transformation** if $A \in \mathcal{A} \implies \mu(T^{-1}(A)) =$	$\mu(T^{-1}(A)) = \mu(A)$ the **size** of *all* points going into A after one iteration is same as the **size** of A
		$\mu\text{–}\mathtt{P}(x \in B) = \mu\text{–}\mathtt{P}(Tx \in B)$ **probability** of picking x which is in B after an iteration $=$ **probability** of picked x in B
	$f : X \to \mathbb{R}$	*observable*
	$U_T(f) := f \circ T : X \to \mathbb{R}$	
	$\mathfrak{x}_T^{(n)} := f \circ T^n : X \to \mathbb{R}$ a stochastic process?	
	$\int_X f$	$\mu\text{–}\mathtt{E}(f)$ *expectation value*
	$\mu(B) = \int_X \chi_B \mathrm{d}\mu$	$\mu(B) = \mu\text{–}\mathtt{E}(\mathtt{check}(- \in B))$
$\chi_B \circ T^k$	$\chi_B \circ T^k(x)$	$\mathtt{check}(T^k(x) \in B) \in \{0,1\}$
		$\mathtt{returns}(B) := \{x \in B \mid \exists n >$
	• Hence we conclude $\mu(A) > 0 \implies \{T^{-n}(A)$ which is $\iff$ there exists $n, m \in \mathbb{N}, n > m$ $x \in T^{-n}(A) \cap T^{-m}(A) =$ that is there exists points in A return back to A.	$\mu(B) > 0 \implies \mathtt{returns}(B) \neq$ or $\mathtt{returns}(B) = \emptyset \implies \mu(B) =$

	• Consider the set $A \subset B$ of points that do not return to B $$A := \{x \in B \mid \forall n > 0 : T$$, so this means points of A do not return to A either. If $T^{-n}(A)$ are *not* pairwise disjoint for all $n \in \mathbb{N}$ then there exists $n, m \in \mathbb{N}, n > m$ $$x \in T^{-n}(A) \cap T^{-m}(A) =$$ meaning the point $T^n(x)$ in A returns to A, which contradicts the definition of A. Thus $T^{-n}(A)$ are all disjoint.	$\texttt{returns}(B \setminus \texttt{returns}(B)) = ($
	(weak form of Poincare recurrence) Let $(X, \mathcal{A}, \mu)$ be a **probability space** and $T : X \to X$ be a μ-preserving transformation and $B \subseteq X$ such that $\mu(B) > 0$. Then the set of points in B that never returns to B has measure zero.	$\mu(B) > 0 \implies \mu(B \setminus \texttt{returns}(B)) = 0$
	(infinite Poincare recurrence) Let $(X, \mathcal{A}, \mu)$ be a **probability space** and $T : X \to X$ be a μ-preserving transformation and $B \subseteq X$ such that $\mu(B) > 0$. Then the set of points in B that returns to B *only finitely many times* has measure zero.	
ergodic	Definition. Ergodic endomorphism A μ-preserving transformation $T : X \to X$ on a measure space $(X, \mathcal{A}, \mu)$ is called μ-**ergodic** if $$A \in \mathcal{A}, T^{-1}(A) = A \implies$$	
strongly mixing	Definition. Strongly mixing endomorphism A μ-preserving transformation $T : X \to X$ on a **probability** space $(X, \mathcal{A}, \mu)$ is called **strongly mixing** if $$A, B \in \mathcal{A} \implies \mu(A \cap T^{-n}$$	$\mu\text{–}\mathrm{P}(x \in A \& T^n x \in B) \to \mu(A$

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  - Measure spaces
    - Measure preserving endomorphisms
      - Interpretation of measure preserving endomorphisms

And it has 2 siblings.

- structures based on sets
  - Measure spaces
    - Measure preserving endomorphisms
      - Interpretation of measure preserving endomorphisms
      - Recurrence in iterations of a measure preserving endomorphism