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Laplacian on a Riemannian manifold

Definition. Laplacian on a Riemannian manifold

Let (M, g) be an oriented Riemannian manifold with/without boundary. Then the **Laplacian** or the **Laplace-Beltrami operator** on (M, g) is the operator

$$\Delta_g : \mathcal{C}^\infty(M) \rightarrow \mathcal{C}^\infty(M)$$
$$u \mapsto \Delta_g u := \operatorname{div}_g(\operatorname{grad}_g(u)) = d_g^\dagger du$$

Proposition:

$$\Delta_g(fh) = f(\Delta_g h) + 2 \langle \operatorname{grad}_g(f), \operatorname{grad}_g(h) \rangle + h(\Delta_g f)$$

Proposition: In local coordinates,

$$\Delta u = \frac{1}{\sqrt{\det g_{ij}}} \partial_k \left(\sqrt{\det g_{ij}} g^{kl} \partial_l u \right)$$

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