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Irreducible representations of S_n

Definition. Definition

Given a Young diagram for the partition

$$\lambda = (\lambda_1, \dots, \lambda_l), \lambda_1 + \dots + \lambda_l = n$$

we number the boxes canonically by the integers $1, \dots, n$. This defines two subgroups of S_n

$$P_\lambda := \{g \in S_n \mid g \text{ preserves each row of the Young tablau of } \lambda\}$$
$$Q_\lambda := \{g \in S_n \mid g \text{ preserves each column of the Young tablau of } \lambda\}$$

In $\mathbb{C}[S_n]$ we define two elements corresponding to these subgroups

$$a_\lambda := \sum_{g \in P_\lambda} g$$
$$b_\lambda := \sum_{g \in Q_\lambda} \text{parity}(g)g$$

Finally we set,

$$c_\lambda := a_\lambda b_\lambda \in \mathbb{C}[S_n]$$

called the **Young symmetrizer** of S_n for the partition λ .

what does the a_λ, b_λ do?

As $S_n \curvearrowright V^{\otimes n}$ by permuting factors for any vector space V , the image of the element

$$a_\lambda \in \mathbb{C}[S_n] \rightarrow \text{EndVec}_{\mathbb{C}}(V^{\otimes n})$$

is the subspace

$$a_\lambda V^{\otimes n} = \bigotimes_{i=1}^l \textbf{Sym}^{\lambda_i} V \hookrightarrow \bigotimes_{i=1}^l V^{\otimes \lambda_i} \hookrightarrow V^{\otimes n}$$

λ	$a_\lambda, b_\lambda, c_\lambda$	image of a_λ on $V^{\otimes n}$	image of b_λ on $V^{\otimes n}$	image of c_λ on $\mathbb{C}[S_n]$ as a rep denoted as V_λ
(n)	$c_{(n)} = a_{(n)} = \sum_{g \in S_n} g$	$c_{(n)} V^{\otimes n} = \textbf{Sym}^n V$		trivial representation
$(1, 1, \dots, 1)$				alternating representation
the conjugate partition to λ , denoted λ'				$V_{\lambda'} \cong V_\lambda \otimes V_{(1, 1, \dots, 1)}$

Current note has 0 direct children and 0 total descendants.

- stretchy
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And it has 1 siblings.

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