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## $Z_{k-1}\text{sing}^\infty(M, \mathbb{Z}) \rightarrow U(1)$ differential characters

Definition.  $Z_{k-1}\text{sing}^\infty(M, \mathbb{Z}) \rightarrow U(1)$  differential characters

A  $Z_{k-1}\text{sing}^\infty(M, \mathbb{Z}) \rightarrow U(1)$  differential  $k$ -character on a smooth manifold  $M$  is a pair

$$(\chi, F)$$

where

$$\chi : Z_{k-1}\text{sing}^\infty(M, \mathbb{R}) \rightarrow U(1)$$

is a group homomorphism and for every  $k$ -chain  $B$  (that is  $B \in Z_k\text{sing}^\infty(M, \mathbb{Z})$ ),

$$\chi(\partial B) = \exp \left( i \int_B F \right)$$

[1]

$$\implies F \in \text{Forms}_{2\pi\mathbb{Z}}^l(M)$$

Definition. Group of all  $Z_{k-1}\text{sing}^\infty(M, \mathbb{Z}) \rightarrow U(1)$  differential characters

Denote the set of all

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as

$$\check{H}^l(M)$$

which is an Abelian group with the addition

$$(\chi_1, F_1) + (\chi_2, F_2) = (\chi_1 + \chi_2, F_1 + F_2)$$

### topologically trivial characters

$A \in \text{Forms}^{l-1}(M)$  and define

$$\chi_A(W) := \exp \left( i \int_W A \right)$$

then

$$A + \omega \in [A] \in \frac{\text{Forms}^{l-1}(M)}{\text{Forms}_{\mathbb{Z}}^{l-1}(M)} \implies \chi_A = \chi_{A+\omega}$$

So we call the elements of

$$\frac{\text{Forms}^{l-1}(M)}{\text{Forms}_{\mathbb{Z}}^{l-1}(M)}$$

as *topologically trivial characters*.

Current note has 0 direct children and 0 total descendants.

- structures based on sets
  - manifolds
    - smooth  $\mathbb{R}$ -manifold
      - Smooth singular homology of a smooth manifold
        - $Z_{k-1}\text{sing}^\infty(M, \mathbb{Z}) \rightarrow U(1)$  differential characters

And it has 2 siblings.

- structures based on sets
  - manifolds
    - smooth  $\mathbb{R}$ -manifold
      - Smooth singular homology of a smooth manifold
        - $Z_{k-1}\text{sing}^\infty(M, \mathbb{Z}) \rightarrow U(1)$  differential characters
        - Dual differential forms of smooth singular chains