Info

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Elementary group with finite and infinite sided Dirichlet polyhedra

Let $\theta \in \mathbb{R}$ be such that ϕ/π is irrational. Then

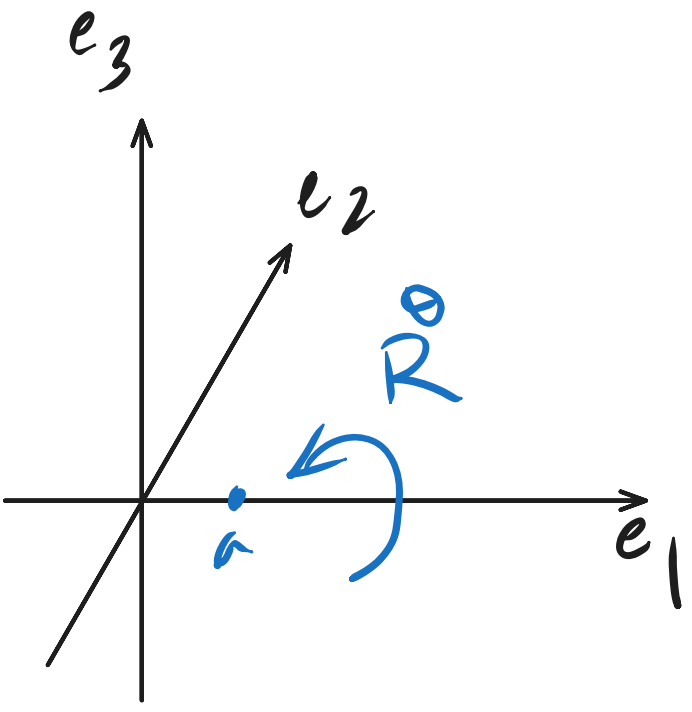
$$A = \begin{bmatrix} 1 & \\ & R^\theta \end{bmatrix}$$

is an irrational rotation of $\mathbb{R}^3$ about the axis e_1. Let $f := e_1 + A$ which is an isometry of $\mathbb{R}^3$ that preserves $\mathbb{R}\{e_1\}$.

Consider the cyclic group

$$\langle f \rangle \leq \mathrm{Isom}(\mathbb{R}^3)$$

> **Proposition:** For $a \in \mathbb{R}\{e_1\}$ consider the closed region between $\mathbb{R}\{e_2, e_3\} \pm \frac{1}{2}e_1$. It is the Dirichlet domain $D(\langle f \rangle, a)$ of $\langle f \rangle \curvearrowright \mathbb{R}^3$ based at a.
> In particular, $\langle f \rangle$ is discrete.



> **Proposition:** For $a \in \mathbb{R}^3 \setminus \mathbb{R}\{e_1\}$, $P(a)$ has infinitely many sides.

- Suppose $P(a)$ has finitely many sides.
  - Let $S \in \mathrm{sides}(P(a))$. Then $\mathrm{plane}(S)$ is the perpendicular bisector of $\overline{a\ f^m a}$ for some integer $m \neq 0$.
  - $\implies$ $\mathrm{plane}(S)$ intersects the line

    $$L := \{(a_1, ta_2, ta_3) | t \in \mathbb{R}\}$$

    passing through a.

[1]

Current note has 0 direct children and 0 total descendants.

- squishy
  - Elementary group with finite and infinite sided Dirichlet polyhedra

And it has 30 siblings.

- squishy
  - $SL(2)(\mathbb{R}) \curvearrowright \mathbb{R}^2$
  - $SL(2)(\mathbb{R}) \curvearrowright H^2_{\mathbb{U}}$
  - $\mathrm{Mat}_{\mathbb{C}}(2)$
  - $A_{r,}\text{-}\langle z \mapsto \lambda z \rangle \backslash \mathbb{R}\boldsymbol{H}^2_{\mathbb{U}}$
  - $\mathbb{C} \setminus \{0\}_{,}\text{-}\langle z \mapsto z+1 \rangle \backslash \mathbb{R}\boldsymbol{H}^2_{\mathbb{U}}$
  - $\mathbb{C} \setminus \{0,1\}_{,}\text{-}PSL(2)(\mathbb{Z})[2] \backslash \mathbb{R}\boldsymbol{H}^2_{\mathbb{U}}$
  - $\mathbb{R} \ltimes_\theta \mathbb{C}^2$
  - $\boldsymbol{Cl}(\mathbb{R}^n, \langle\ ,\ \rangle_{0,n})$
  - $\mathbb{C}^n$
  - $U(\mathbb{C} \setminus \{0\})$
  - $PSL(2)(\mathbb{C}) \curvearrowright \mathbb{C}P^1 \cong_{\mathsf{Man}} S^2$
  - $\mathbb{C}P^n$
  - Farey graph, complex and tree
  - GL
  - Heisenberg group
  - Incomplete hyperbolic polygon mod side-pairing
  - $\mathbb{R}\boldsymbol{H}^2_{,}\text{-}\mathbb{R}H^2_{\mathbb{U},}\text{-}D^2$
  - $\mathbb{R}\boldsymbol{H}^n$
  - D_n lattice
  - Matrices
  - $\mathrm{Mat}_{\mathbb{R}}(2)$
  - Mobius endomorphisms
  - Elementary group with finite and infinite sided Dirichlet polyhedra

- Polynomials
- $T^*S^1 \ncong_{\mathsf{Sym}} (\mathbb{R}^2 \setminus \{0\}, \mathrm{d}x \wedge \mathrm{d}y)$
- $Z\left(\sum_{1 \leq j \leq n} z_j^2 - 1\right)(\mathbb{C}) \cong_{\mathsf{Sym}} T^*S^{n-1}$
- SL
- SO
- Seifert–Weber dodecahedral 3-manifold
- $U(n, \mathbb{C})$

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1. John G. Ratcliffe - Foundations of Hyperbolic Manifolds (2019), p.642 ↩